METHOD FOR DETECTING HIDDEN PERIODIC COMPONENTS WITHIN SEISMICITY.

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INTRODUCTION.

Methodologically, analysis of seismic catalogs is more difficult than processing of such traditional sources of geophysical information as time series (derived from seismic observations or low-frequency geophysical monitoring). This is due to the fact that the analysis of point processes [Cox, Lewis, 1966], including earthquakes sequences, does not allow the direct application of a vast variety of methods, parametrical models, and fast algorithms developed in the theory of signals [Brillinger, 1975; Marple (Jr.), 1987; Hannan, 1970]. Actually, application of these methods requires a preliminary conversion of seismic catalogs to time series, which are sequences of values with a given constant time step. Formally, this conversion is not difficult and can be realized via calculation of either average values of a certain catalog parameter (for example, energy released during an earthquake) in successive non-overlapping time windows of a constant width or cumulative values of these characteristics with a constant time step (cumulative curves). However, the resulting time series are essentially non-Gaussian and include either outliers or step-like features (in cumulative curves) due to the time non-uniformity of seismic catalogs (gaps and groups of events such as swarms and aftershocks) and concentrating of major seismic energy in rare but strong events (the well-known problem of "heavy tails" of distributions). Although classical methods of the signal analysis, based on the Fourier transformation and calculating of covariances, are formally applicable to the processing of these time series, they are ineffective due to large biases in estimates caused by outliers (or steps).

Even such a routine operation as estimation of a power spectrum and extraction of periodical components from a signal gives ambiguous in the analysis of seismic catalogs and requires the development of special approaches based on evaluation of model parameters of a point process [Vere-Jones and Ozaki, 1982; Lyubushin *et al*., 1998]. Joint analysis of several catalogs aggravates the computation and mathematical difficulties. Elaborating (for the multidimensional case) the class of interaction models for point processes, proposed by Ogata and Akaike [1982] and Ogata *et al*. [1982], Lyubushin and Pisarenko [1993] developed a parametric method to quantify the influence of the seismic regime in several regions on the seismicity of a given region.

Below the method elaborated in [Lyubushin *et al*., 1998] for investigating periodic components in seismic process will be modified for application in moving time window and will be applied for several case studies.

THE METHOD.

Let

$$\mathbf{I_k = [T_k^{(0)}, T_k^{(1)}] \qquad k=1,\ldots,m} \tag{1}$$

be a sequence of non-overlapping time intervals of seismic events registration. The gaps between these time intervals could be caused by different external reasons usually by wars. We shall consider events which have magnitude values above the given threshold:

$$\mathbf{M \ge M_0} \tag{2}$$

Let us consider the following model of seismic intensity which has a periodic component:

$$\boldsymbol{\lambda_k(t) = \mu_k \cdot (1 + a \cdot \cos(\omega t + \varphi))} \quad \text{for} \quad \mathbf{t \in I_k} \tag{3}$$

where the frequency ω, amplitude $\mathbf{a, 0 \le a \le 1}$ and phase angle φ, $\varphi \in \mathbf{[0,2\pi]}$ are common for all time intervals $\mathbf{I_k}$ whereas multipliers $\mu_k \ge \mathbf{0}$ (which describe a Poissonian part of seismic process intensity within time interval $\mathbf{I_k}$) are individual for each interval. Thus, the Poissonian part of intensity is modulated by harmonic oscillation.

Let $\mathbf{N_k}$ be a general number of events within $\mathbf{I_k}$, satisfying the condition (2). Let us fix some value of the frequency ω. Logarithmic likelihood function for the whole set of observations is equal to the sum of log-likelihood functions of each interval:

$$\mathbf{\ln L(\vec{\mu}, a, \varphi \mid \omega) = \sum_{k=1}^{m} \ln L_k(\mu_k, a, \varphi \mid \omega)} \tag{4}$$

where $\vec{\mu} = (\mu_1, \ldots, \mu_m)$,

$$\begin{aligned} \mathbf{\ln L_k(\mu_k, a, \varphi \mid \omega)} &= \mathbf{\sum_{t_i \in I_k} \ln(\lambda_k(t_i)) - \int_{I_k} \lambda_k(s) ds =} \\ &= \mathbf{N_k \ln(\mu_k) + \sum_{t_i \in I_k} \ln(1 + a\cos(\omega t_i + \varphi)) - \mu_k \Delta T_k -} \\ &- \mathbf{\frac{\mu_k a}{\omega}[\sin(\omega T_k^{(1)} + \varphi) - \sin(\omega T_k^{(0)} + \varphi)]} \end{aligned} \tag{5}$$

where $\mathbf{\Delta T_k = T_k^{(1)} - T_k^{(0)}}$. Taking maximum value of (5) with respect to μ_k it is easily to find that

$$\mathbf{\mu_k = \mu_k(a, \varphi \mid \omega) = N_k \Big/ [\Delta T_k + \frac{a}{\omega}[\sin(\omega T_k^{(1)} + \varphi) - \sin(\omega T_k^{(0)} + \varphi)]]} \tag{6}$$

Thus

$$\ln(L_k(\mu_k, a, \varphi \mid \omega)) = \sum_{t_i \in I_k} \ln(1 + a\cos(\omega t_i + \varphi)) + N_k \ln(\mu_k(a, \varphi \mid \omega) - N_k \tag{7}$$

It should be noted that $\mu_k(a = 0, \varphi \mid \omega) = N_k / \Delta T_k \equiv \mu_k^{(0)}$ is the estimate of the Poissonian part of intensity within I_k.

Thus, the increment of log-likelihood function due to introduction of the harmonic oscillation with given frequency value ω into the model of intensity with respect to zero hypothesis that seismic process is pure random (Poissonian) is

$$\Delta \ln L(a, \varphi \mid \omega) = \sum_{k=1}^{m} \Delta \ln L_k(a, \varphi \mid \omega) \tag{8}$$

where

$$\Delta \ln L_k(a, \varphi \mid \omega) = \sum_{t_i \in I_k} \ln(1 + a\cos(\omega t_i + \varphi)) + N_k \ln(\frac{\mu_k(a, \varphi \mid \omega)}{\mu_k^{(0)}}) \tag{9}$$

Let

$$R(\omega) = \max_{a,\varphi} \Delta \ln L(a, \varphi \mid \omega), \; 0 \le a \le 1, \; \varphi \in [0, 2\pi], \tag{10}$$

The function (10) could be regarded as the generalization of the spectra for the sequence of events. The graphic of this function indicates which probe values of the frequency provide the maximum gain in log-likelihood function increment with respect to a pure random model. Thus, the points of maximum of the function (10) detect periodic components of the seismic process.

The next generalization of this approach is estimating the function (10) not over the whole time interval of observation but within moving time window of the certain length T_W. Let τ be a time coordinate of the right-hand end of the moving time window. Then we have the function of 2 arguments: $R(\omega, \tau \mid T_W)$ which could be visualized as 2D map within the plane of (ω, τ)-values. The time-frequency diagrams allow describe the dynamics of periodic component within seismic process.

CASE STUDIES.

I present a number of graphics of the function (10) and 2D time-frequency maps $\mathbf{R}(\omega,\tau\,|\,\mathbf{T_W})$ for global seismic process. The data were taken from the site http://wwwneic.cr.usgs.gov/neis/. For presenting results it is more convenient to use periods instead of frequency values.

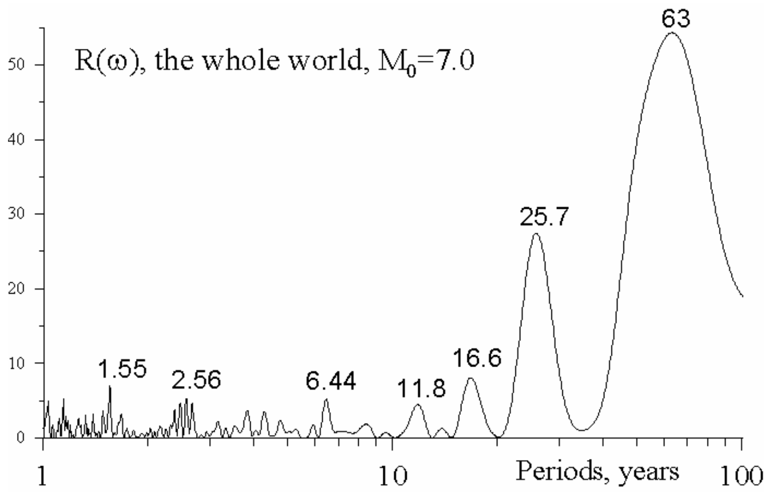

Fig.1. The static estimate for the whole world, 1901-2001, M_0=7.0, depth ≤ 100 km.

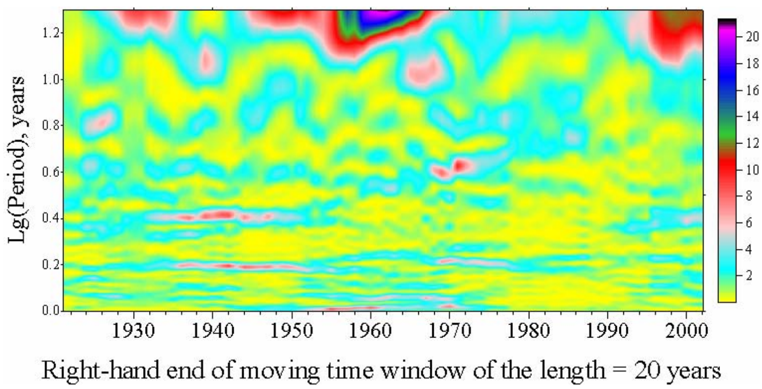

Fig.2. The time-frequency estimate for the whole world, 1901-2001, M_0=7.0, depth ≤ 100 km.

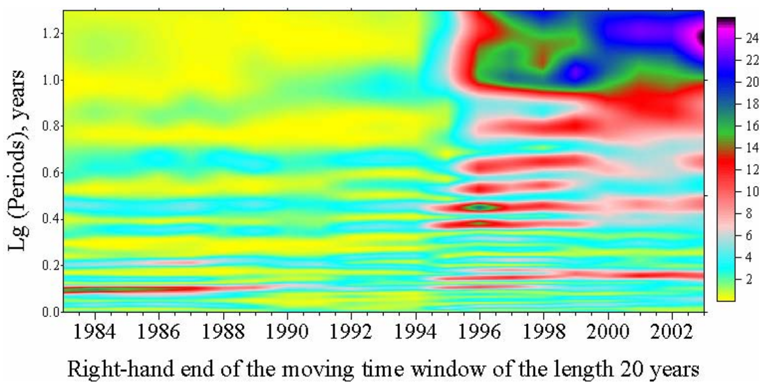

Fig.3. The time-frequency estimate for the region: (Japan + Kuril Islands + South Kamchatka), 1963-2003, M_0=6.0, depth ≤ 100 km.

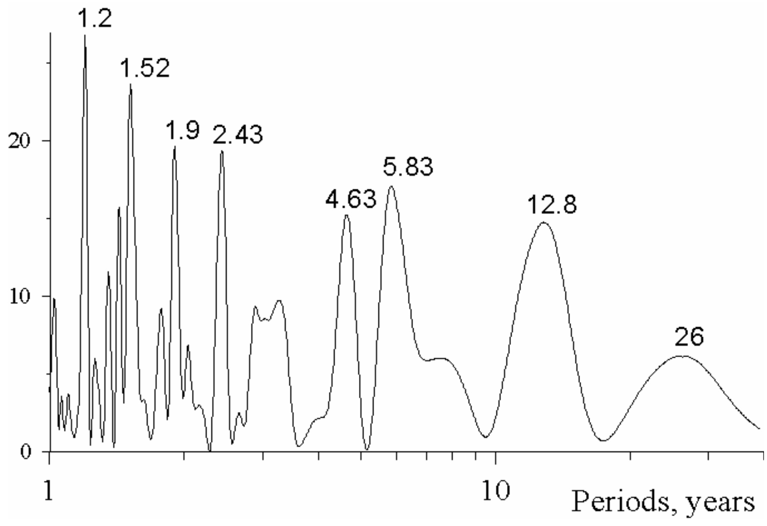

Fig.4. The static estimate for California, 1963-2001, M_0=4.5, depth ≤ 100 km.

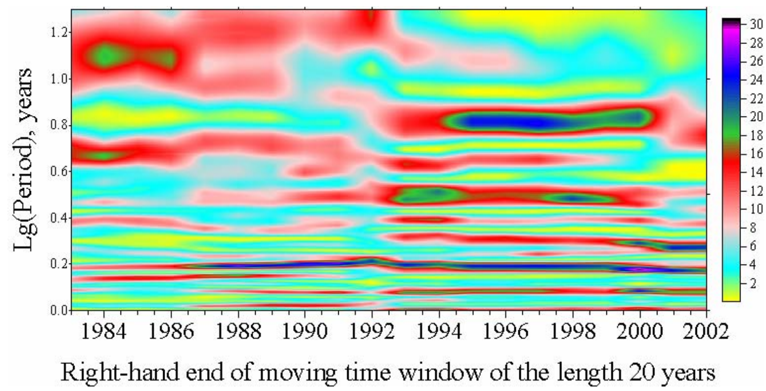

Fig.5. The time-frequency estimate for California, 1963-2001, M_0=4.5, depth ≤ 100 km.

CONCLUSION.

The elaborated method provides the insight view into the dynamic periodic structure of the seismic process which could be used for seeking new precursors of strong earthquakes as the migration of the main period of harmonic component.

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