Seismic Regimes Interaction Analysis by Influence Matrices Method.

Alexey A. Lyubushin

Institute of the Physics of the Earth, Russian Academy of Sciences,
123995, Moscow, Russia, Bolshaya Gruzinskaya, 10;
fax : +007-095-255-60-40; e-mail: lubushin@mtu-net.ru

We will divide the considerable seismically active region into **m**≥2 nonintersecting sub regions. Let

$$\{ \mathbf{t}_j^{(\alpha)}, \mathbf{M}_j^{(\alpha)} \}, \mathbf{j}=1,\ldots, \mathbf{N}_\alpha ; \ \alpha=1,\ldots,\mathbf{m}$$

be a seismic catalog of α -*th* sub region, **m** is the whole number of sub regions. Here $\mathbf{t}_j^{(\alpha)}$ are time moments of earthquakes, $\mathbf{M}_j^{(\alpha)}$ - their values of magnitude. Let's denote by $\lambda^{(\alpha)}(\mathbf{t})$ an intensity (number of shocks per time unit) of seismic process in the region "α". We shall use the following model of intensity:

$$\lambda^{(\alpha)}(\mathbf{t}) = \mu^{(\alpha)} + \sum_{\beta=1}^{\mathbf{m}} \mathbf{b}_\beta^{(\alpha)} \cdot \mathbf{g}^{(\beta)}(\mathbf{t}) \qquad (1)$$

where $\mu^{(\alpha)} \geq \mathbf{0}, \mathbf{b}_\beta^{(\alpha)} \geq \mathbf{0}$ - parameters, $\mathbf{g}^{(\beta)}(\mathbf{t})$ - function of influence of β -*th* region. We use the following model of such functions:

$$\mathbf{g}^{(\beta)}(\mathbf{t}) = \sum_{\mathbf{t}_j^{(\beta)}<\mathbf{t}} \exp(\mathbf{r} \cdot (\mathbf{M}_j^{(\beta)} - \mathbf{M}_0)) \cdot \exp(-(\mathbf{t} - \mathbf{t}_j^{(\beta)})/\tau) \qquad (2)$$

Here **r**≥0, τ >0 are parameters of the model, $\mathbf{M}_0$ is a cutoff magnitude value (minimal value to be considered in the analysis). Thus, according to formula (2), weight of the **j**-*th* event becomes nonzero for the moments $\mathbf{t} > \mathbf{t}_j^{(\beta)}$, increases exponentially with its magnitude $\mathbf{M}_j^{(\beta)}$ and exponentially decay (with characteristic time τ) when time **t** increases from the moment of the earthquake $\mathbf{t}_j^{(\beta)}$. The sum of all such diminishing time response functions for time events rigorously less than the current time value **t** forms the general influence function $\mathbf{g}^{(\beta)}(\mathbf{t})$ of β -*th* region. The $\mathbf{b}_\beta^{(\alpha)}$ parameter are scaling factors and their values, in essence, determine the degree of influence of β -*th* region on α -*th* (such influence we will denote by the sign "$\beta \to \alpha$"). The $\mathbf{b}_\alpha^{(\alpha)}$ values determine the degree of influence of the α -*th* region on itself. As for the parameter $\mu^{(\alpha)}$, its values reflects the contribution of the purely Poissonian component to the process intensity, for which an "influence function" is constant and equals 1.

Let fix for a while some values of the parameters $(\mathbf{r}, \tau)$ and consider a problem for choosing values of scaling factors parameters $(\mu^{(\alpha)}, \mathbf{b}_\beta^{(\alpha)})$. We will use the maximum likelihood method. The logarithmic likelihood function for non-stationary point process for α -*th* region could be written in the following form [*Cox, Lewis, 1966; Cox, 1975*]:

$$\ln(L_\alpha) = \sum_{j=1}^{N_\alpha} \ln(\lambda^{(\alpha)}(t_j^{(\alpha)})) - \int_0^T \lambda^{(\alpha)}(s)ds \tag{3}$$

where $[0,T]$ is the time interval of observation. Thus it is necessary to find the maximum of the function (3) with respect to parameters $(\mu^{(\alpha)}, b_\beta^{(\alpha)})$. From necessary conditions of maximum with respect to $(\mu^{(\alpha)}, b_\beta^{(\alpha)})$ it follows that:

$$\mu^{(\alpha)} \frac{\partial \ln(L_\alpha)}{\partial \mu^{(\alpha)}} + \sum_{\beta=1}^{m} b_\beta^{(\alpha)} \frac{\partial \ln(L_\alpha)}{\partial b_\beta^{(\alpha)}} = N_\alpha - \int_0^T \lambda^{(\alpha)}(s)ds \tag{4}$$

Because parameters $(\mu^{(\alpha)}, b_\beta^{(\alpha)})$ must be non-negative each of the term in the left-hand part of (4) becomes equal to zero at the point of maximum of function (3) because if the parameter is positive, the corresponding partial derivative becomes equal or zero, but if the maximum is attained at the boundary, the values of the parameter becomes equal to zero. Consequently, the following condition is satisfied at the point of the maximum of the log-likelihood function:

$$\int_0^T \lambda^{(\alpha)}(s)ds = N_\alpha \tag{5}$$

Let's substitute expression (1) into (5) and divide by the length of the observational interval. Then we obtain another form of (5):

$$\mu^{(\alpha)} + \sum_{\beta=1}^{m} b_\beta^{(\alpha)} \cdot \bar{g}^{(\beta)} = \mu_0^{(\alpha)} \equiv \frac{N_\alpha}{T} \tag{6}$$

where $\bar{g}^{(\beta)} = \frac{1}{T}\int_0^T g^{(\beta)}(s)ds$ - a mean value of the influence function. Substituting $\mu^{(\alpha)}$ from (6) to (3), we'll obtain the following maximization problem, which is equivalent to (3):

$$\Phi^{(\alpha)}(b_1^{(\alpha)},...,b_m^{(\alpha)}) = \sum_{j=1}^{N_\alpha} \ln(\mu_0^{(\alpha)} + \sum_{\beta=1}^{m} b_\beta^{(\alpha)} \cdot \Delta g^{(\beta)}(t_j^{(\alpha)})) \to \max \tag{7}$$

where $\Delta g^{(\beta)}(t) = g^{(\beta)}(t) - \bar{g}^{(\beta)}$, under the restrictions:

$$b_1^{(\alpha)} \geq 0, \ldots, b_m^{(\alpha)} \geq 0, \quad \sum_{\beta=1}^{m} b_\beta^{(\alpha)} \cdot \bar{g}^{(\beta)} \leq \mu_0^{(\alpha)} \tag{8}$$

It is easy to show that function (7) is convex, its Hessian is negatively determined and thus the problem (7), (8) has a unique solution.

We have fixed parameters (r, τ) until. The reason for this is that the problem (3) is multi-extremal and not regular with respect to parameters (r, τ) [*Ogata et all, 1982*]. That is why we vary parameters (r, τ) on a regular lattice of their values:

$$\mathbf{r} = \{ \mathbf{r_p}, \mathbf{p} = \mathbf{0}, \ldots, \mathbf{L_r} \}, \ \mathbf{r_p} = \mathbf{r_{min}} + \Delta\mathbf{r} \cdot \mathbf{p}, \Delta\mathbf{r} = (\mathbf{r_{max}} - \mathbf{r_{min}}) / \mathbf{L_r} \tag{9}$$

$$\tau = \{\tau_\mathbf{q}, \mathbf{q} = \mathbf{0}, \ldots, \mathbf{L}_\tau\}, \tau_\mathbf{q} = \tau_\mathbf{min} + \Delta\tau \cdot \mathbf{q}, \Delta\tau = (\tau_\mathbf{max} - \tau_\mathbf{min}) / \mathbf{L}_\tau \tag{10}$$

and for each pair $(\mathbf{r}, \tau)$ we solve the problem (7), (8) and after all choose those values of $(\mathbf{r}, \tau)$ for which the corresponding maximum value of (7) is the maximal.

Having solved the problem (7), (8) with the “best” pair $(\mathbf{r}, \tau)$ for each region, we can introduce a main object of investigation - a matrix $\{ \kappa_\beta^{(\alpha)}, \alpha = \mathbf{1}, \ldots, \mathbf{m}; \beta = \mathbf{0}, \mathbf{1}, \ldots, \mathbf{m} \}$ with components:

$$\kappa_\mathbf{0}^{(\alpha)} = \frac{\mu^{(\alpha)}}{\mu_\mathbf{0}^{(\alpha)}} \geq \mathbf{0}, \quad \kappa_\beta^{(\alpha)} = \frac{\mathbf{b}_\beta^{(\alpha)} \cdot \overline{\mathbf{g}}^{(\beta)}}{\mu_\mathbf{0}^{(\alpha)}} \geq \mathbf{0} \tag{11}$$

which we’ll name *an influence matrix*. Interpretation of its components is rather transparent: $\kappa_\mathbf{0}^{(\alpha)}$ is the part (share) of the mean intensity of α *-th* region, which is pure stochastic; $\kappa_\beta^{(\alpha)}$ - a share, which is caused by the influence $\beta \to \alpha$. From the formula (5) it follows that:

$$\kappa_\mathbf{0}^{(\alpha)} + \sum_{\beta=1}^{\mathbf{m}} \kappa_\beta^{(\alpha)} = \mathbf{1} \tag{12}$$

In order to investigate the influence effects between different seismoactive regions, we’ll estimate influence matrix (11) not over all time interval [0,T] of observation, but independently in overlapping moving time windows.

References.

Cox D.R., Lewis P.A.W. The statistical analysis of series of events. 1966. London, Methuen.
Cox D.R. Partial likelihood. Biometrica. 1975, v.62, 269-276.
Lyubushin A.A.(Jr), V.F.Pisarenko. Research on seismic regime using linear model of intensity of interacting point processes - Fizika Zemli (Solid Earth), 1993, N12, 81-87.
Ogata Y., H.Akaike. On linear intensity models for mixed doubly stochastic Poisson and self-exciting point processes. Journal of the Royal Statistical Society. Series B (Methodological), 1982a, v.44, No.1, 102-107.
Ogata Y., H.Akaike, K.Katsura. The application of linear intensity models to the investigation of causal relations between a point process and another stochastic process. Annals of the Institute of Statistical Mathematics, 1982b, v.34, No.2, Pt.B, 373-387.