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Application of a Bayesian Approach for Estimation of Seismic Hazard Parameters in Some Regions of the Circum-Pacific Belt

T. M. Tsapanos,[1] A. A. Lyubushin,[2] and V. F. Pisarenko[3]

Abstract—The maximum possible (regional) magnitude M_{max} and other seismic hazard parameters like β which is the slope of Gutenberg-Richter law, and λ which is the intensity (rate) of seismic activity are estimated in eight seismic regions of the west side of the circum-Pacific belt. The Bayesian approach, as described by (Pisarenko *et al.*, 1996; Pisarenko and Lyubushin, 1997, 1999) is a straightforward technique of estimating the seismic hazard. The main assumptions for the method applied are a Poissonian character of seismic events flow, a frequency-magnitude law of Gutenberg-Richter's type with cutoff maximum value for the estimated parameter and a seismic catalog, which have a rather sizeable number of events. We also estimated the quantiles of the probabilistic distribution of the "apparent" M_{max} for future given time-length intervals.

Key words: Bayesian approach, maximum possible magnitude, quantiles of magnitude distribution, circum-Pacific belt.

1. Introduction and Data

A large number of models are currently available for the assessment of seismic hazard. The objective in seismic hazard modeling is to obtain long-term probabilities of occurrence of seismic events of specific size in a given time interval. One of the main inconsistencies in the seismic hazard assessment is the estimation of the maximum magnitude and the related uncertainty. The "apparent" magnitude (Tinti and Mulargia, 1985; Kijko and Sellevoll, 1992) which represents the observed magnitude $\bar{M}$, is equal to the "true" magnitude M plus an uncertainty ε. The probability distribution of this uncertainty can be modeled by various distribution functions.

[1] Aristotle University of Thessaloniki, School of Geology, Geophysical Laboratory, 54006 Thessaloniki, Greece. E-mail: tsapanos@geo.auth.gr

[2] Academy of Sciences of Russia, United Institute of the Physics of the Earth, Russia, 123810, Moscow, Bolshaya Gruzinskaya, 10, Russia. E-mail: lubushin@sirius.mitp.ru also: lubushin@uipe-ras.scgis.ru

[3] Academy of Sciences of Russia, International Institute for Earthquake Prediction, Russia, 113556, Moscow, Varshavskoye sh., 79, Russia. E-mail: vlad@sirius.mitp.ru

The theory of Bayesian probability expresses the formulation of the inferences from data straightforward and allows the solution of problems which otherwise would be intractable. Assuming the Poisson model, BENJAMIN (1968) was the first to deal with the Bayesian approach to investigate the problem of earthquake occurrence. MORTGAT and SHAH (1979) presented a Bayesian model for seismic hazard mapping, which takes into account the geometry of the faults in the investigated area, while CAMPBELL (1982, 1983) proposed a Bayesian extreme value distribution of earthquake occurrence to evaluate the seismic hazard along the San Jacinto fault. A similar procedure has been applied by STAVRAKAKIS and TSELENTIS (1987) for a probabilistic prediction of strong earthquakes in Greece.

FERRAES (1985, 1986) used a Bayesian analysis to predict the interarrival times for strong earthquakes along the Hellenic arc, as well as for Mexico. An alternative view of Ferraes research is made by PAPADOPOULOS (1987) for the occurrence of large shocks in the east and west sides of the Hellenic arc. Recently STAVRAKAKIS and DRAKOPOULOS (1995) adopted the Bayesian extreme-value distribution of earthquake occurrence in order to estimate the seismic hazard in certain seismogenic zones in Greece and the surrounding area. An effort was made by LAMARRE *et al.* (1992) to make a realistic evaluation of seismic hazard.

For the purpose of the present work, the earthquakes with magnitude $M \geq 7.0$ are considered and the events listed in the catalogue of PACHECO and SYKES (1992) are taken into account. In order to extend the data set-up to 1996 earthquakes are extracted from the bulletins of N.E.I.C. Main shocks only are analyzed. This study is restricted to shallow ($h \leq 60$ km) earthquakes only between 1900–1996.

An effort is made in the present study to estimate the seismic hazard parameters and their uncertainties based on a Bayesian estimation procedure, proposed by PISARENKO *et al.* (1996), and generalizes in PISARENKO and LYUBUSHIN (1997, 1999) applying to the maximum seismic peak ground acceleration problem. This approach is applied to the real data as recorded in some of the most seismoactive regions of the circum-Pacific belt, and the aim is to illustrate its function in various seismotectonic environments.

2. Method Applied

Now we shall present the main points of the method, following PISARENKO *et al.* (1996), and PISARENKO and LYUBUSHIN (1997, 1999). Let R be some value, which was measured or estimated as a sequence on a "past" time interval $(-\tau, 0)$:

$$\vec{R}^{(n)} = (R_1, \ldots, R_n),\ R_i \geq R_0, \quad R_\tau = \max_{1 \leq i \leq n}(R_1, \ldots, R_n) \ . \tag{1}$$

The values (1) could be of arbitrary physical nature. Below we shall consider (1) as earthquakes' magnitudes in a given seismoactive region. R_0 is a minimum cutoff value, i.e., a value defined by possibilities of registration systems or a minimum value up from which the value sequence (1) is statistically representative. We use letter "R", not "M", although later we shall consider the earthquakes' magnitudes only, in order to underline that the used method can be applied to any problems which estimate maximum values, where the recurrence law of the Gutenberg-Richter type can be written.

The first assumption for applying the method is that values (1) obey the Gutenberg-Richter law of distribution:

$$\mathrm{Prob}\{R < x\} = F(x|R_0, \rho, \beta) = \frac{e^{-\beta \cdot R_0} - e^{-\beta \cdot x}}{e^{-\beta \cdot R_0} - e^{-\beta \cdot \rho}}, \quad R_0 \le x \le \rho \ . \tag{2}$$

Here ρ is the unknown parameter which represents the maximum possible value of R, for instance, maximum possible value of earthquake magnitudes in a given region. The unknown parameter β is usually called the "slope" of the Gutenberg-Richter law at small values of x when the dependence (eq. 2) is plotted in doubly logarithmic axes.

The second assumption is that the sequence (eq. 1) is a Poissonian process with some intensity value λ, which is also an unknown parameter.

Thus the full vector of the unknown parameter is the following:

$$\theta = (\rho, \beta, \lambda) \ . \tag{3}$$

Let ε be an error for magnitudes, with which we know values (eq. 1), i.e., for us actually in (eq. 1) are accessible not *true*, but *apparent* values of R, which are defined by the formula:

$$\bar{R} = R + \varepsilon \ . \tag{4}$$

Let $n(x|\delta)$ be a density of probabilistic distribution of the error ε, where δ is a given scale parameter of the density. We shall use the following uniform distribution density:

$$\begin{aligned} n(x|\delta) &= \frac{1}{2\delta}, \quad |x| \le \delta \\ n(x|\delta) &= 0, \quad |x| > \delta \ . \end{aligned} \tag{5}$$

Let $\bar{f}(R|\theta, \delta)$ be the probability density for apparent magnitudes values, $\bar{F}(R|\theta, \delta)$ its cummulative distribution function and $\bar{\lambda}(\theta, \delta)$ the intensity of apparent magnitudes. Then

$$\bar{f}(x|\theta, \delta) = \frac{1}{(c_f A_1 - A_2)} \cdot \widehat{f} \ , \tag{6}$$

where $\widehat{f} = c_f \beta A(x)$, for $R_0 \le x < \rho - \delta$; $\widehat{f} = \frac{A(x-\delta) - A_2}{2\delta}$, for $\rho - \delta \le x \le \rho + \delta$;

$$A(x) = \exp(-\beta x), \quad A_1 = A(R_0), \quad A_2 = A(\rho),$$

$$c_f = c_f(\theta, \delta) = \frac{\exp(\beta\delta) - \exp(-\beta\delta)}{2\delta}$$

and

$$\bar{F}(x|\theta,\delta) = \frac{1}{(c_f A_1 - A_2)} \widehat{F} \quad , \tag{7}$$

where $\widehat{F} = c_f(A_1 - A(x))$, for $R_0 \leq x < \rho - \delta$.

$$\widehat{F} = c_f(A_1 - A(\rho-\delta)) - A_2 \cdot \frac{(x-\rho+\delta)}{2\delta} - \frac{[A(x-\delta) - A(\rho - 2\delta)]}{2\beta\delta}, \quad \text{for } \rho - \delta \leq x \leq \rho + \delta \ ;$$

$$\bar{\lambda}(\theta,\delta) = \lambda \cdot c_f(\theta,\delta) \ . \tag{8}$$

The derivation of formulas (6), (7), (8) for the case (5) can be found in KIJKO and SELLEVOLL (1992).

Let Π be *a priori* uncertainty domain of values of parameters θ

$$\Pi = \{\lambda_{\min} \leq \lambda \leq \lambda_{\max},\ \beta_{\min} \leq \beta \leq \beta_{\max},\ \rho_{\min} \leq \rho \leq \rho_{\max}\} \ . \tag{9}$$

We shall consider the *a priori* density of the vector θ to be uniform in the domain Π.

Let [0, T] be a future interval of time for which we want to estimate the distribution function of the maximum value ρ and its quantiles.

Since the flow of events (eq. 1) is stationary and Poissonian, the intensity of events with $R < x$ equals $\lambda \cdot F(x|\theta)$ and the intensity of events with $R \geq x$ equals $\lambda \cdot (1 - F(x|\theta))$. From the Poissonian character of the events flow (eq. 1) it follows that the probability that no events on time interval [0, T] will have $R \geq x$ or that all events on [0, T] will have $R < x$ equals:

$$\exp(-\lambda \cdot (1 - F(x|\theta)) \cdot T) \ . \tag{10}$$

Let us denote by R_T the maximal value of R on the time interval [0, T]. Then $\text{Prob}\{R_T < x\} = \exp(-\lambda \cdot (1 - F(x|\theta)) \cdot T)$. However included in this probability is the case when there is no event on [0, T]. Let us denote by v_T the number of events with $R \geq R_0$ on the interval [0, T]. Then

$$\text{Prob}\{v_T = 0\} = e^{-\lambda \cdot T}; \quad \text{Prob}\{v_T \geq 1\} = (1 - e^{-\lambda \cdot T}) \ . \tag{11}$$

That is why

$$\begin{aligned} \Phi_T(x|\theta) &= \text{Prob}\{R_T < x | v_T \geq 1\} \\ &= \frac{\exp(-\lambda T(1 - F(x|\theta)) - \exp(-\lambda T)}{1 - \exp(-\lambda T)} \\ &= \frac{\exp(\lambda T F(x|\theta)) - 1}{\exp(\lambda T) - 1} \ . \end{aligned} \tag{12}$$

Formula (12) defines an expression for the *a priori* distribution function of the *true* maximum values of R on the future time interval $[0, T]$. Let us introduce also the following functions

$$\phi_T(x|\theta) = \frac{d}{dx}\Phi_T(x|\theta) \tag{13}$$

— the *a priori* density for the *true* maximum values of R on time interval $[0, T]$;

$$Y_T(\alpha|\theta) \text{ – the root of equation: } \Phi_T(x|\theta) = \alpha, \quad 0 \le \alpha \le 1 \tag{14}$$

— the *a priori* quantile for probability α for the *true* maximum values of R on time interval [0, T]; quantile of the random value ξ of probability α means a minimum root of the equation: $\mathrm{Prob}\{\xi < x\} = \alpha$ (see KENDALL *et al.*, 1987).

If we substitute in formula (8) $F(x|\theta) \to \bar{F}(x|\theta, \delta)$ then we will obtain a function:

$\bar{\Phi}_T(x|\theta, \delta)$ — the *a priori* function of distribution for *apparent* maximum values of R on future time interval $[0, T]$.

Substituting $\bar{\Phi}_T(x|\theta, \delta)$ into formulas (11) and (12), we obtain:

$\bar{\phi}_T(x|\theta, \delta)$ — the *a priori* density for the *apparent* maximum values of R on the future time interval $[0, T]$, and

$\bar{Y}_T(\alpha|\theta, \delta)$ — the *a priori* quantile for probability α for the *apparent* maximum values of R on the future time interval $[0, T]$.

According to the definition of conditional probability, the *a posteriori* density of distribution of the vector of parameters θ is equal to:

$$f(\theta|\vec{R}^{(n)}, \delta) = \frac{f(\theta, \vec{R}^{(n)}|\delta)}{f(\vec{R}^{(n)}|\delta)} \tag{15}$$

but $f(\theta, \vec{R}^{(n)}|\delta) = f(\vec{R}^{(n)}|\theta, \delta) \cdot f^a(\theta)$, where $f^a(\theta)$ is the *a priori* density of the distribution of vector θ in the domain Π. As $f^a(\theta) = \mathrm{const}$ according to our assumption and taking into consideration that

$$f(\vec{R}^{(n)}|\delta) = \int\limits_{\Pi} f(\vec{R}^{(n)}|\theta, \delta) d\theta \ , \tag{16}$$

then we will obtain after using a Bayes formula (RAO, 1965) and normalizing the density that

$$f(\theta|\vec{R}^{(n)}, \delta) = \frac{f(\vec{R}^{(n)}|\theta, \delta)}{\int\limits_{\Pi} f(\vec{R}^{(n)}|\vartheta, \delta) d\vartheta} \ . \tag{17}$$

In order to use eq. (17) we must have an expression for the function $f(\vec{R}^{(n)}|\theta, \delta)$. With the assumption of Poissonian character of the sequence in eq. (1) and of independence of its members, we can obtain:

$$f(\vec{R}^{(n)}|\theta, \delta) = \bar{f}(R_1|\theta, \delta) \cdots \bar{f}(R_n|\theta, \delta) \cdot \frac{\exp(-\bar{\lambda}(\theta, \delta) \cdot \tau) \cdot (\bar{\lambda}(\theta, \delta) \cdot \tau)^n}{n!} \ . \tag{18}$$

Now we are ready to compute a Bayesian estimate of vector θ:

$$\hat{\theta}(\vec{R}^{(n)}|\delta) = \int_{\Pi} \vartheta \cdot f(\vartheta|\vec{R}^{(n)}, \delta) d\vartheta \ . \tag{19}$$

One of the components of this vector (eq. 19) contains an estimate of maximum value ρ. Using a formula analogous to eq. (19), we can obtain Bayesian estimates of any of the functions (eqs. 12, 13 and 14). The most interesting for us are estimates of quantiles of distribution functions of true and apparent R values on a given future time interval $[0, T]$, for instance for α quantiles of apparent values

$$\hat{\bar{Y}}_T(\alpha|\vec{R}^{(n)}, \delta) = \int_{\Pi} \bar{Y}_T(\alpha|\vartheta, \delta) \cdot f(\vartheta|\vec{R}^{(n)}, \delta) d\vartheta \ . \tag{20}$$

$\hat{Y}_T(\delta|\vec{R}^{(n)}, \delta)$ for α quantiles of true values is written analogously to eq. (20). Using averaging over the density (eqs. 17 and 18) we can also estimate variances of Bayesian estimates (eqs. 19 and 20). For example

$$\mathrm{var}\{\hat{\bar{Y}}_T(\alpha|\vec{R}^{(n)}, \delta)\} = \int_{\Pi} (\bar{Y}_T(\alpha|\vartheta, \delta) - \hat{\bar{Y}}_T(\alpha|\vec{R}^{(n)}, \delta))^2 \cdot f(\vartheta|\vec{R}^{(n)}, \delta) d\vartheta \ . \tag{21}$$

In order to finish the description of the method, we must define the domain of *a priori* uncertainty Π (eq. 9).

Firstly we set $\rho_{\min} = R_\tau - \delta$. As for the value of $\rho_{\max}$, it is introduced by the user of the method and depends on the specifics of the data series (1). For instance, for the estimation of maximum magnitudes in Japan we put $\rho_{\max} = 9.5$. Boundary values for the slope β are defined by the formula

$$\beta_{\min} = \beta_0 \cdot (1 - \gamma), \quad \beta_{\max} = \beta_0 \cdot (1 + \gamma), \quad 0 < \gamma \le 1 \tag{22}$$

where β_0 is the "central" value, obtained as a maximum likelihood estimate of the slope for the Gutenberg-Richter law:

$$\sum_{i=1}^{n} \ln\left\{\frac{\beta \cdot e^{-\beta \cdot R_i}}{e^{-\beta \cdot R_0} - e^{-\beta \cdot R_\tau}}\right\} \to \max_{\beta, \beta \in (0, \beta_s)} \tag{23}$$

where β_s is a rather large value, for example 10, and value γ is a parameter of the method which usually is $\gamma = 0.5$.

For setting boundary values for the intensity in eq. (9) we use the following reasons. As a consequence of normal approximation for a Poissonian process for rather large n (Cox and Lewis, 1966) the standard deviation of the value $\lambda\tau$ has the approximate value $\sqrt{n} \approx \sqrt{\lambda\tau}$. Therefore, taking boundaries at $\pm 3 \cdot \sigma$, we will obtain:

$$\lambda_{\min} = \lambda_0 \cdot \left(1 - \frac{3}{\sqrt{\lambda_0 \tau}}\right), \quad \lambda_{\max} = \lambda_0 \cdot \left(1 + \frac{3}{\sqrt{\lambda_0 \tau}}\right) \tag{24}$$

where

$$\lambda_0 = \frac{\bar{\lambda}_0}{c_f(\beta_0, \delta)}, \quad \bar{\lambda}_0 = \frac{n}{\tau} \ .$$

3. Application of the Method and Results

The method is applied to the west side of the circum-Pacific belt. Most of the seismic regions of this part are of the most seismically active regions of the world. Accordingly we shall examine Alaska and Aleutian Islands (1), Kamchatka (2), Japan (3), Mariane Islands (4), Philippine Islands (5), Indonesia (6), Guinea-Solomon and New Hebrides Islands (7), and Kermadec-Tonga and Fiji Islands (8). In Figure 1 the eight examined seismic regions are illustrated and their division is after TSAPANOS (1990). The digits in brackets refer to the number of each region in accord with Figure 1.

In earlier papers (PISARENKO *et al.*, 1996; PISARENKO and LYUBUSHIN, 1997, 1999) the above method was applied in order to estimate maximum values of magnitudes and seismic peak ground accelerations, their functions of distribution and quantiles for a number of regions in California, Italy and Caucasus.

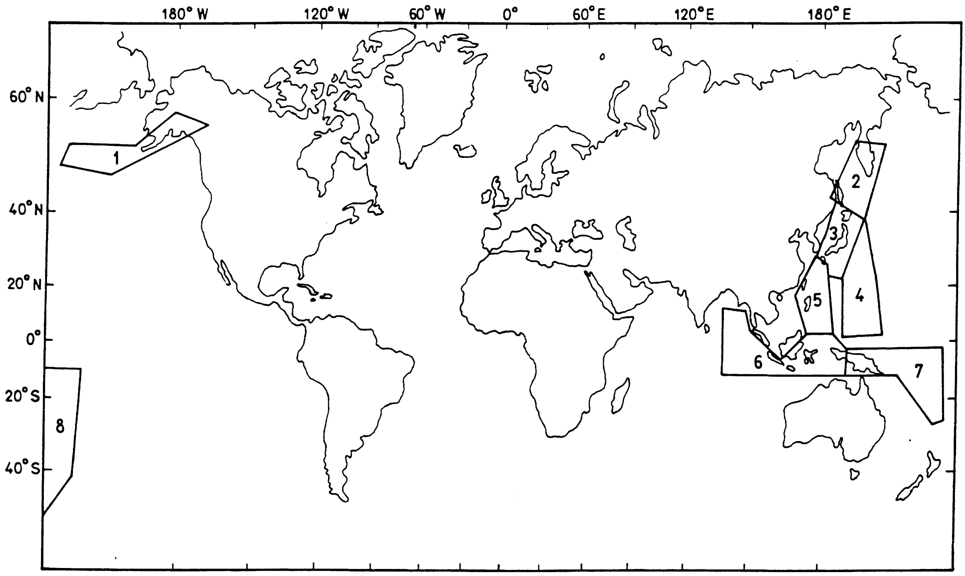


Figure 1
The eight examined seismic regions of the world (after TSAPANOS, 1990). According to the numbering of these regions are: (1) Alaska and Aleutian Islands, (2) Kamchatka, (3) Japan, (4) Mariane Islands, (5) Philippine Islands, (6) Indonesia, (7) New Guinea-Solomon Islands-New Hebrides Islands, (8) Kermadec-Tonga-Fiji Islands.

In Table 1 the estimated maximum possible M_{max} (regional) magnitude, the slope β of the magnitude frequency relation and the intensity λ (rate) with their standard deviations (eq. 21) are listed. The value of δ – the scale parameter of the noise distribution (eq. 5) was taken 0.2 for all variants.

Emphasis is placed on the estimation of the maximum possible (regional) magnitude M_{max}, as well the quantiles of the M_{max} distribution in a future time interval. The M_{max} estimation through the Bayesian approach is comparable with the M_{max} obtained by TSAPANOS (2001). The mean difference between the M_{max} of the two methods is 0.25, which means that Bayesian method estimates the M_{max} slightly larger than the maximum likelihood approach. The standard deviations obtained in the above-analyzed method are reasonable, varying between 0.26 and 0.38. The Bayesian procedure is more stable but is a more time consuming method (PISARENKO *et al.*, 1996). The maximum observed apparent magnitude $\bar{M}_{max}$ is also tabulated for comparison purposes. Another annotation relates that the method provides the mean "apparent" intensity of shocks, as well as the "true" value of mean intensity (shocks/day) which is the one written in Table 1, and the reference of the slope β means the estimation of the slope which is also listed in Table 1. For instance, we estimated for Alaska and the Aleutians the mean "apparent" intensity as 0.0017 (shocks/day), while the "true" mean intensity is equal to 0.0016, because it varies between 0.00092 and 0.0022. It is similar for slope β, which has boundaries for uncertainty domain Π between 1.98 and 5.95 for the same region. This happens because the procedure considers a different β and λ for a different cut-off of magnitudes in each step. Thus this model takes into account a different number of earthquakes in different parts of the magnitude-frequency relation – a significant number of earthquakes exist for the low magnitudes, and there are less events in the large magnitudes. Even in our case where we analyzed earthquake, with magnitude $M \geq 7.0$, we must have different slope in the very large magnitudes ($M \geq 8.0$).

Table 1

The estimates of the Bayesian analysis in the seismic regions of the west Pacific. The tabulation shows the estimates of maximum possible magnitude M_{max}, the β parameter and the intensity λ (rate) in events per day, and their uncertainties. The M_{max} as obtained by TSAPANOS (2001) are in brackets. Also the maximum observed apparent magnitude $\bar{M}_{max}$ and the number N of earthquakes with $M \geq 7.0$ that occurred during the examined period are also listed

Region Name	N	M_{max} ± st.dev.	$\bar{M}_{max}$	β ± st.dev.	λ ± st.dev.
Alaska-Aleutian	58	8.89 ± 0.34 (8.46)	8.4	4.08 ± 0.54	0.0016 ± 0.00021
Kamchatka	55	8.80 ± 0.38 (8.50)	8.4	3.57 ± 0.49	0.0015 ± 0.00020
Japan	100	8.69 ± 0.36 (8.69)	8.6	2.97 ± 0.35	0.0027 ± 0.00026
Mariane Islands	15	7.96 ± 0.30 (7.89)	7.6	5.82 ± 1.26	0.00039 ± 0.00010
Philippine Islands	90	8.47 ± 0.29 (8.16)	8.1	4.64 ± 0.51	0.0023 ± 0.00025
Indonesia	77	8.39 ± 0.33 (8.05)	8.0	4.87 ± 0.58	0.0019 ± 0.00022
Guinea-Solomon-Hebrid	176	8.42 ± 0.30 (8.12)	8.1	4.73 ± 0.38	0.0043 ± 0.00033
Kermadec-Tonga-Fiji	51	8.46 ± 0.26 (8.29)	8.2	2.92 ± 0.48	0.0015 ± 0.00021

The *a posteriori* probability density for the apparent and true (Figs. 2a, b) M_{max} magnitude, as well as the *a posteriori* probability distribution function for the apparent and true (Figs. 3a, b) M_{max} magnitudes that will occur in a future time interval of 5, 10, 20, 50 and 100 years are determined for Alaska and the Aleutian Islands, only as an example. Both figures are useful probabilistic tools for seismic hazard estimation. For all of the analyzed regions, quantiles (apparent) are estimated and graphs of their distribution are prepared, the Aleutian-Alaska, Kamchatka, Japan, Mariane Islands depicted in Figure 4, while the Philippine Islands, Indonesia, New Guinea-Solomon-New Hebrides Islands, and Kermadec-Tonga-Fiji Islands illustrated in Figure 5. In Figures 4 and 5 the quantiles of the level of probability $\tau = 0.50$ (medians) and $\alpha = 0.90$ are depicted. Both quantiles of apparent and true magnitudes can be estimated and are illustrated in Tables 2 and 3.

If we compare Tables 2 and 3 it is easy to observe that the values in Table 3 are less than those of Table 2. This is because Table 2 includes magnitudes which are

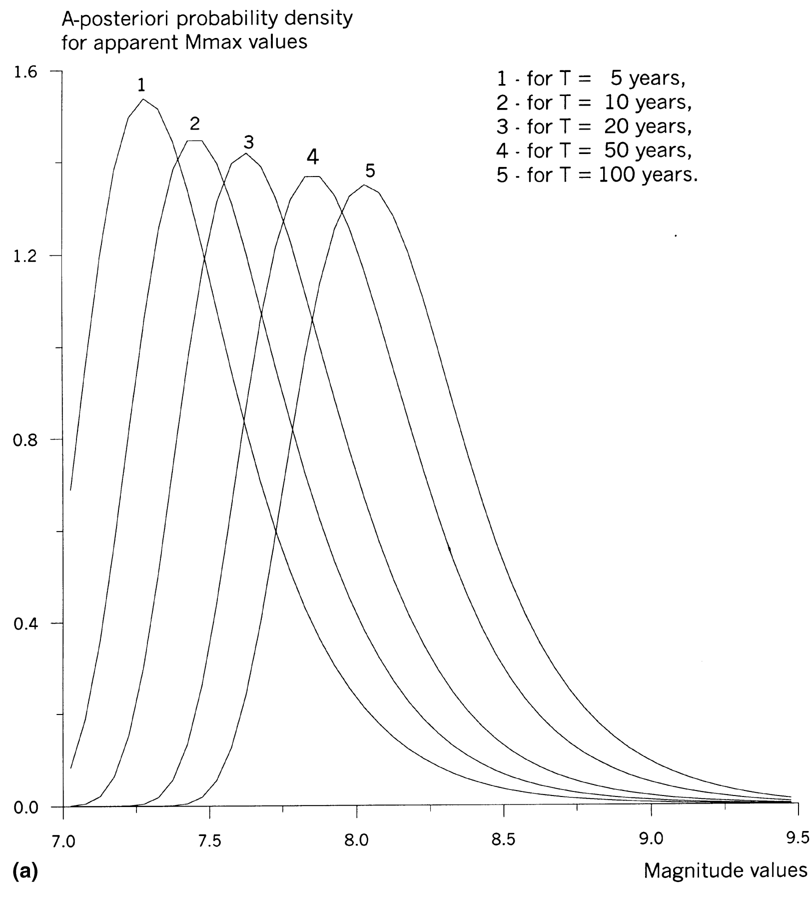


Figure 2a

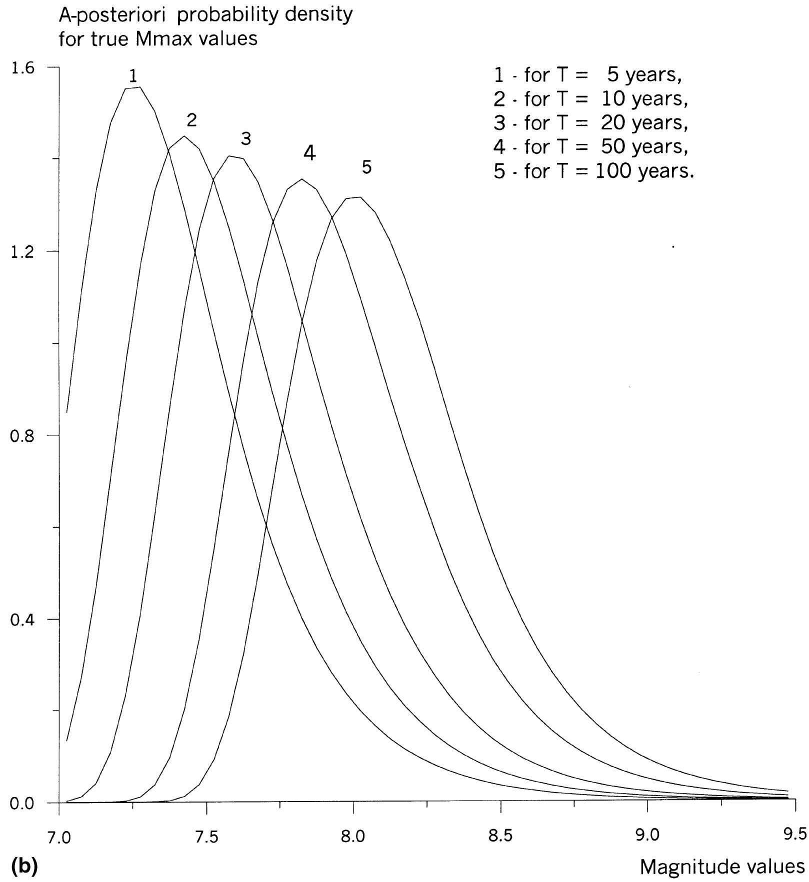


Figure 2
Statistical characteristics of seismic hazard parameters for Alaska and Aleutian Islands. *A posteriori* probability densities of $M_{max}(T)$, where $T = 5, 10, 20, 50$ and 100 next years for: a) apparent magnitude, and b) true magnitude.

these of Table 3 plus the error ε. The difference is very low and we believe that this depends on the quality of the data, which include minor errors. Therefore the quality of the data included in the Tables 2 and 3 is almost the same.

4. Discussion and Conclusions

An efficient Bayesian approach is applied in the present paper in order to test this special model on data from areas with different seismotectonic regimes. The estimates of M_{max} through this method, are comparable to other estimates obtained by different approaches. It differs from the M_{max} obtained by TSAPANOS (2001) by

0.25 orders of magnitude. It is also greater by 0.34 than the maximum apparent magnitude $\bar{M}_{max}$. The Bayesain approach needs an *a priori* distribution for unknown parameters. Nonetheless the dependence on the *a posteriori* estimators to the *a priori* distribution is negligible for a "big" sample. Only large, mainly shallow earthquakes are considered for this analysis. This method is applicable for a uniform distribution of magnitudes, although the Gaussian distribution could also be used (KIJKO and SELLEVOLL, 1992).

Other related parameters that can be estimated through this Bayesian approach are the slope β of the Gutenberg-Richter magnitude-frequency relation, as well as the intensity λ of the seismic events occurring per day. According to PISARENKO *et al.* (1996), these parameters are reasonably estimated if we use an instrumental catalogue of earthquakes for a period of 50 years or more. The length of our catalogue is approximately 100 years, thus we believe that the estimations are accurate.

For Alaska and the Aleutian islands we obtained the *a posteriori* probability density and the *a posteriori* probability distribution function for both the "apparent"

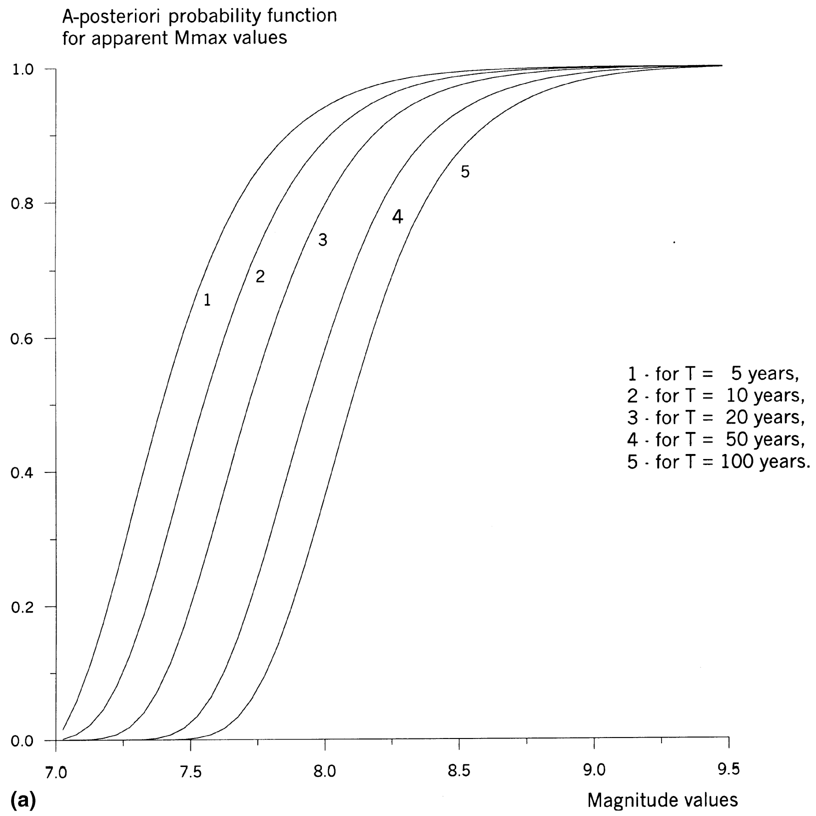


Figure 3a

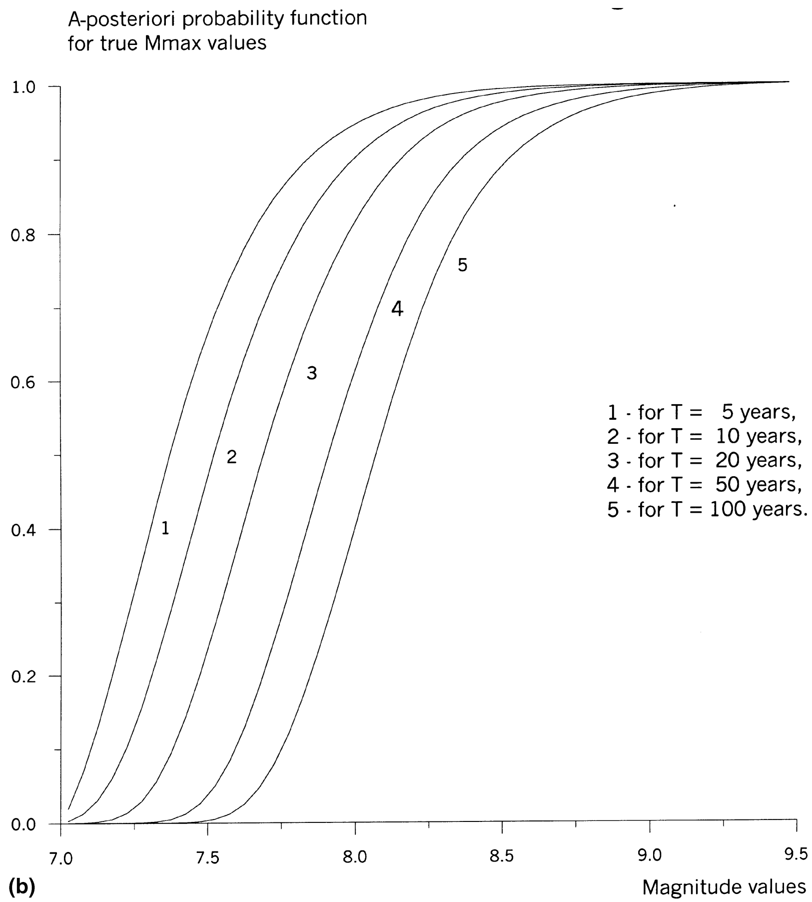


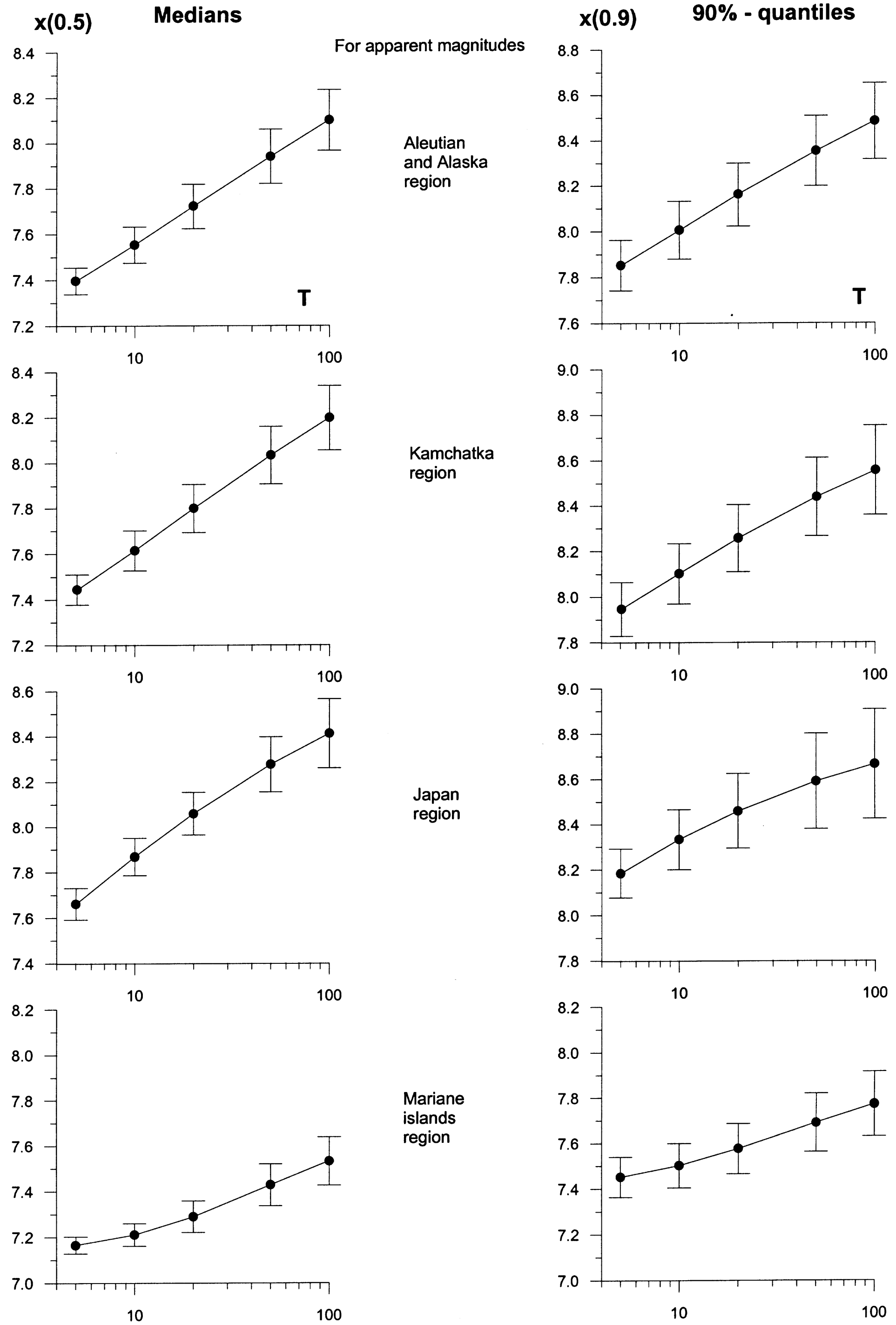
Figure 3
Statistical characteristics of seismic hazard parameters for Alaska and Aleutian Islands. *A posteriori* probability functions of $M_{max}(T)$, where $T = 5, 10, 20, 50$ and 100 next years for: a) apparent magnitude, and b) true magnitude.

and "true" M_{max} values that will occur in the future time interval of 5, 10, 20, 50 and 100 years.

Finally, the *a posteriori* M quantiles are estimated for the eight examined regions and for probabilities of 0.50, 0.60, 0.70, 0.80, 0.90, 0.95 and 0.98. Only two of them are plotted – the medians-quantile, which is of level of probability 0.50, and the 90%-quantile, for future times T which correspond to 5, 10, 20, 50 and 100 years. Their

▶

Figure 4
Quantiles for apparent magnitudes (of medians and 90%) of the distribution function of maximum values of M_{max} for a given length T of future time interval for the seismic regions of: Alaska and Aleutian Islands, Kamchatka, Japan, Mariane Islands. The standard deviation intervals are designated by vertical bars.


x(0.5)
Medians
For apparent magnitudes
x(0.9)
90% - quantiles
Aleutian
and Alaska
region
Kamchatka
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Japan
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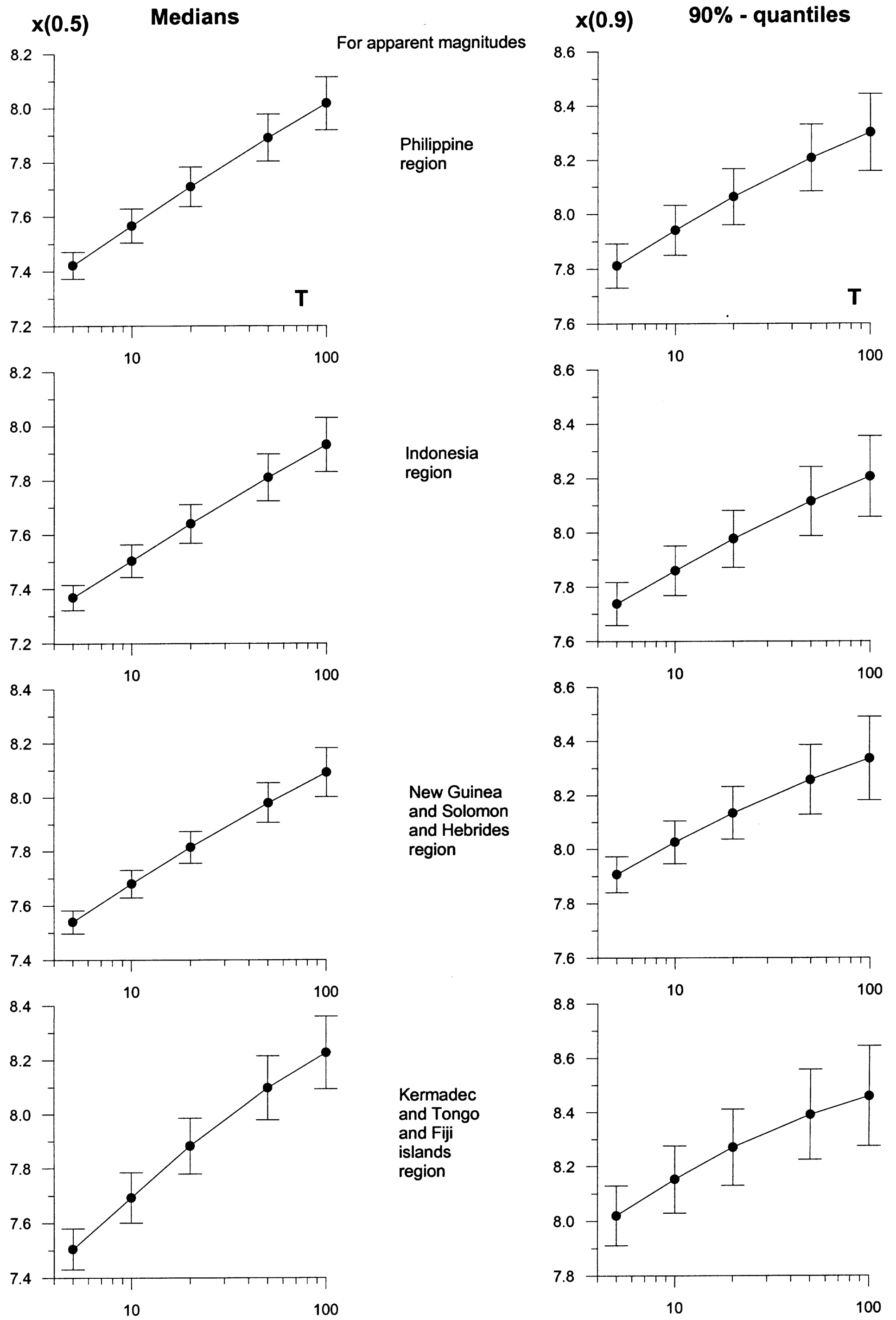
x(0.5)
Medians
For apparent magnitudes
x(0.9)
90% - quantiles
Philippine region
Indonesia region
New Guinea and Solomon and Hebrides region
Kermadec and Tongo and Fiji islands region
T
T
10
100

Table 2

The estimated quantiles of the apparent magnitudes $M_{\max}(T)$, where $T = 5, 10, 20, 50, 100$ are the lengths of the future time interval in years, for the levels of probability $\alpha = 0.50$ and $\alpha = 0.90$ for the eight examined seismic zones of the west Pacific area. The regions referred to are in the same order as those in Table 1

	Quantiles of probability level 0.50					Quantiles of probability level 0.90				
Years	5	10	20	50	100	5	10	20	50	100
Region										
1	7.40	7.55	7.72	7.94	8.10	7.85	8.00	8.26	8.35	8.48
2	7.44	7.61	7.80	8.03	8.20	7.94	8.10	8.25	8.44	8.55
3	7.66	7.87	8.06	8.28	8.41	8.18	8.33	8.46	8.59	8.67
4	7.16	7.21	7.29	7.43	7.53	7.45	7.50	7.58	7.69	7.77
5	7.42	7.57	7.71	7.89	8.02	7.81	7.94	8.06	8.21	8.30
6	7.37	7.50	7.64	7.81	7.93	7.74	7.86	7.98	8.11	8.20
7	7.54	7.68	7.81	7.98	8.09	7.91	8.02	8.13	8.25	8.33
8	7.50	7.69	7.88	8.10	8.22	8.02	8.15	8.27	8.39	8.46

Table 3

The estimated quantiles of the true magnitudes $M_{\max}(T)$ where *$T = 5, 10, 20, 50, 100$ are the lengths of the future time interval in years, for the levels of probability $\alpha = 0.50$ and $\alpha = 0.90$ for the eight examined seismic zones of the west Pacific area. The regions referred to are in the same order as those in Table 1*

	Quantiles of probability level 0.50					Quantiles of probability level 0.90				
Years	5	10	20	50	100	5	10	20	50	100
Region										
1	7.37	7.52	7.69	7.91	8.07	7.83	7.98	8.13	8.32	8.45
2	7.42	7.59	7.78	8.01	8.17	7.93	8.07	8.23	8.41	8.52
3	7.64	7.84	8.04	8.26	8.38	8.16	8.31	8.43	8.54	8.60
4	7.15	7.19	7.26	7.39	7.49	7.43	7.48	7.54	7.64	7.71
5	7.39	7.53	7.68	7.86	7.98	7.78	7.91	8.03	8.17	8.25
6	7.34	7.47	7.61	7.78	7.90	7.71	7.83	7.94	8.07	8.16
7	7.51	7.65	7.78	7.95	8.06	7.87	7.99	8.10	8.21	8.27
8	7.49	7.67	7.86	8.08	8.20	8.00	8.13	8.24	8.34	8.39

confidence limits are computed as well. The estimated quantiles for both "apparent" and "true" magnitudes $M_{\max}(T)$ are determined for 0.50 and 0.90 levels of probability and are tabulated. Their difference is very limited and we believe that this depends on the good quality of the data we used.

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Figure 5

Quantiles for apparent magnitudes (of medians and 90%) of the distribution function of maximum values of $M_{\max}$ for a given length T years of future time interval for the seismic regions of: Philippine, Indonesia, New Guinea-Solomon Islands-New Hebrides Islands, Kermadec-Tonga-Fiji Islands. The standard deviation intervals are denoted by vertical bars.

In the present paper we have shown how this Bayesian model provides a rational methodology for evaluating the future seismic hazard. The structure of the model is such that it can handle any quality and quantity of information in a consistent manner.

A question could arise as to why we use uniform distribution but not normal, for instance? The reason is the simplicity of computing integrals in formulas (19)–(21). The normal distribution for the errors was tested for some examples also, but it produce approximately the same results, especially for the cases of rather considerable values of N (number of events). In general there is no *a priori* advantage for using the normal distribution instead of the uniform one for magnitudes' errors except the consideration that normal is the ''more generally accepted.''

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References

Benjamin, J. R. (1968), *Probabilistic Models for Seismic Forces Design*, J. Struct. Div., ASCE 94, 5T5, 1175–1196.

Campbell, K. W. (1982), *Bayesian Analysis of Extreme Earthquake Occurrences. Part I. Probabilistic Hazard Model*, Bull. Seismol. Soc. Am. *72*, 1689–1705.

Campbell, K. W. (1983), *Bayesian Analysis of Extreme Earthquake Occurrences. Part II. Application to the San Jacinto Fault Zone of Southern California*, Bull. Seismol. Soc. Am. *73*, 1099–1115.

Cox, D. R., and Lewis, P. A. W., *The Statistical Analysis of Series of Events* (London, Methuen, 1966).

Ferraes, S. G. (1985), *The Bayesian Probabilistic Predictions of Strong Earthquakes in the Hellenic Arc*. Tectonophysics *111*, 339–354.

Ferraes, S. G. (1986), *Bayes Theorem and Probabilistic Prediction of Inter-arrival Times for Strong Earthquakes felt in Mexico City*, J. Phys. Earth *34*, 71–83.

Kendall, M. G., Stuart, A., and Ord, J. K., *The Advanced Theory of Statistics, Volume 1: Distribution Theory*, 5th edit. (Oxford University Press, New York, 1987).

Kijko, A., and Sellevoll, M. A. (1989), *Estimation of Earthquake Hazard Parameters from Incomplete Data Files. Part I: Utilization of Extreme and Complete Catalogs with Different Threshold Magnitudes*, Bull. Seismol. Soc. Am. *79*, 645–654.

Kijko, A., and Sellevoll, M. A. (1992), *Estimation of Earthquake Hazard Parameters from Incomplete Data f Files. Part II. Incorporation of Magnitude Heterogeneity*, Bull. Seismol. Soc. Am. *82*, 120–134.

Lamarre, M., Townshed, B., and Shah, H. C. (1992), *Application of the Bootstrap Method to Quantify Uncertainty in Seismic Hazard Estimates*, Bull. Seismol. Soc. Am. *82*, 104–119.

Mortgat, C. P., and Shah, H. C. (1979), *A Bayesian model for seismic hazard mapping*, Bull. Seismol. Soc. Am. *69*, 1237–1251.

Pacheco, J. F., and Sykes, L. R. (1992), *Seismic Moment Catalog of Large Shallow Earthquakes, 1900 to 1989*, Bull. Seismol. Soc. Am. *82*, 1306–1349.

Papadopoulos, G. A. (1987), *An Alternative View of the Bayesian Probabilistic Prediction of Strong Shocks in the Hellenic Arc*, Tectonophysics *132*, 311–320.

PISARENKO, V. F., LYUBUSHIN, A. A., LYSENKO, V. B., and GOLUBEVA, T. V. (1996), *Statistical Estimation of Seismic Hazard Parameters: Maximum Possible Magnitude and Related Parameters*. Bull. Seismol. Soc. Am. *86*, 691–700.

PISARENKO, V. F., and LYUBUSHIN, A. A. (1997), *Statistical Estimation of Maximal Peak Ground Acceleration at a Given Point of Seismic Region*, J. Seismol. *1*, 395–405.

PISARENKO, V. F., and LYUBUSHIN, A. A. (1999), *A Bayesian Approach to Seismic Hazard Estimation: Maximum Values of Magnitudes and Peak Ground Accelerations*, Earthq. Res. in China (English edition), 13, No. 1, pp. 45–57.

RAO, C. R., *Linear Statistical Inference and its Application* (New York, John Wiley, 1965).

STAVRAKAKIS, G. N., and TSELENTIS, G. A. (1987), *Bayesian Probabilistic Prediction of Strong Earthquakes in the Main Seismogenic Zones of Greece*, Boll. Geofis. Teor. Applic. *113*, 51–63.

STAVRAKAKIS, G. N., and DRAKOPOULOS, J. (1995), *Bayesian Probabilities of Earthquake Occurrences in Greece and Surrounding Areas*, Pure Appl. Geophys. *144*, 307–319.

TINTI, S., and MULARGIA, F. (1985), *Effects of Magnitude Uncertainties in the Gutenberg-Richter Frequency-magnitude Law*, Bull. Seismol. Soc. Am. *75*, 1681–1697.

TSAPANOS, T. M. (1990), *b Values of Two Tectonic Parts in the Circum-Pacific Belt*. Pure Appl. Geophys. *134*, 229–242.

TSAPANOS, T. M. (2001), *Evaluation of the Seismic Hazard Parameters for the Selected Regions of the World: The Maximum Regional Earthquake*, Ann. di Geofis., vol. 44 (in press).

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