Multiple Spectral Coherence Measure based on Canonical Coherences

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Multiple spectral coherence measure was suggested in [Lyubushin, 1998] for multidimensional time series processing in the problems of geophysical monitoring. In the papers [Lyubushin, 1999; Lyubushin et al., 2003, 2004; Lyubushin, Sobolev, 2006; Sobolev, Lyubushin, 2007; Lyubushin, 2009, 2010; Lyubushin, Klyashtorin, 2012; Lyubushin, 2014] this spectral measure was applied to different problems of multidimensional time series analysis in geophysics, meteorology, hydrology and climate sciences.

Canonical coherences are the generalization of usual squared coherence spectrum between two scalar time series for the case when two vector time series are considered: m-dimensional time series $X(t)$ and n-dimensional time series $Y(t)$. Here t is an integer time index. Without loss of generality let us suppose that $n \le m$. Squared maximum canonical coherence $\rho_1^2(\omega)$ between multiple time series $X(t)$ and $Y(t)$ is computed as maximum eigenvalue of the following frequency-dependent matrix [Brillinger, 1975; Hanan, 1970]:

$$U(\omega) = S_{xx}^{-1/2} S_{xy} S_{yy}^{-1} S_{yx} S_{xx}^{-1/2} \tag{1}$$

Here ω is the frequency, $S_{xx}(\omega)$ is spectral matrix of the size $m \times m$ of time series $X(t)$, $S_{xy}(\omega)$ is cross-spectrum matrix of the size $m \times n$ between time series $X(t)$ and $Y(t)$, $S_{yx}(\omega) = S_{xy}^H(\omega)$, "$H$" is the sign of Hermitian conjunctions, $S_{yy}(\omega)$ is spectral matrix of the size $n \times n$ of time series $Y(t)$. The value of $\rho_1^2(\omega)$ is used instead of usual squared coherence spectrum when 2 scalar time series are regarded, i.e. when $m = n = 1$.

Let us introduce the notion of by-component canonical coherence $\nu_i^2(\omega)$ of q-dimensional time series $Z(t)$ as squared maximum canonical coherences when in the formula (1) $Y(t)$ is the scalar i-th component of q-dimensional time series $Z(t)$ whereas $X(t)$ is $(q-1)$-dimensional time series composed of all other scalar components of $Z(t)$. Thus, in the formula (1) $n = 1$, $m = (q-1)$.

The value $\nu_i^2(\omega)$ is the measure of connection of variations of the i-th component of q-dimensional time series $Z(t)$ with variations of all other scalar components of $Z(t)$ at the frequency ω. The inequality $0 \le |\nu_i(\omega)| \le 1$ is fulfilled, and the closer the value of $|\nu_i(\omega)|$ to unity, the stronger the linear relation of variations at the frequency ω of the i-th scalar series to analogous variations in all other series. Now we can define the multiple spectral coherence measure by formula:

$$\lambda(\omega) = \prod_{i=1}^{q} |\nu_i(\omega)| \tag{2}$$

The value (2) provides a frequency-dependent measure of linear joint synchronization of variations of all scalar components of time series $Z(t)$ at the frequency ω.

For calculating the measure (2) it is necessary to estimate spectral matrix $S_{zz}(\omega)$ of $Z(t)$ of the size $q \times q$. For this purpose we use vector autoregression model [Marple, Jr., 1987]:

$$Z(t) + \sum_{k=1}^{p} A_k \cdot Z(t-k) = e(t) \tag{3}$$

where t is p an autoregression order, A_k are matrices of autoregression coefficients of the size $q \times q$, $e(t)$ is q-dimensional residual signal with zero mean and covariance matrix $\Phi = M\{e(t)e^T(t)\}$ of the size $q \times q$. Matrices A_k and Φ are defined using Durbin-Levinson procedure and the spectral matrix is calculated using formula:

$$S(\omega) = F^{-1}(\omega) \cdot \Phi \cdot F^{-H}(\omega), \quad F(\omega) = E + \sum_{k=1}^{p} A_k \cdot \exp(-i\omega k) \tag{4}$$

where E is a unit matrix of the size $q \times q$.

When $q = 2$ the value (2) equals to usual squared coherence spectrum:

$$\lambda(\omega) = |S_{12}(\omega)|^2 \big/ \left(S_{11}(\omega) \cdot S_{22}(\omega)\right) \tag{5}$$

where $S_{11}(\omega)$ and $S_{22}(\omega)$ are diagonal elements of the matrix (4), i.e. parametric estimates of the power spectra of two signals, and $S_{12}(\omega)$ is their mutual cross-spectrum.

Let us consider moving time window of the certain length and let τ be the time coordinate of right-hand end of moving time window. If the function (2) will be estimated within each time window independently then we will have time-frequency function:

$$\lambda(\tau, \omega) = \prod_{i=1}^{q} |\nu_i(\tau, \omega)| \tag{6}$$

The value (6) presents the evolution of linear synchronization measure for multiple time series $Z(t)$. Usually some preliminary operations are fulfilled within each time window before estimating spectral matrix (4), such as removing linear trend and coming to increments.

Reference.

Brillinger D.R. (1975) Time series. Data analysis and theory. Holt, Rinehart and Winston, Inc., N.Y., Chicago, San Francisco.

Hannan E.J. (1970) Multiple time series. John Wiley and Sons, Inc., N.Y., London, Sydney, Toronto.

Lyubushin A.A. (1998) Analysis of Canonical Coherences in the Problems of Geophysical Monitoring – Izvestiya, Physics of the Solid Earth, vol.34, 1998, pp. 52-58.
http://alexeylyubushin.narod.ru/Canonical_Coherences_in_the_Problems_of_Geophysical_Monitoring.pdf

Lyubushin A.A. (1999) Analysis of Multidimensional Geophysical Monitoring Time Series for Earthquake Prediction – Annali di Geofisica, Vol.42, No.5, pp.927-937.
http://www.annalsofgeophysics.eu/index.php/annals/article/view/3757

Lyubushin A.A., Pisarenko V.F., Bolgov M.V. and Rukavishnikova T.A. (2003) Study of General Effects of Rivers Runoff Variations – Russian Meteorology and Hydrology, No.7, pp. 59-68.
http://alexeylyubushin.narod.ru/Coherence_measures_of_rivers_runoff_time_series.pdf

Lyubushin A.A., Pisarenko V.F., Bolgov M.V., Rodkin M.V., and Rukavishnikova T.A. (2004) Synchronous Variations in the Caspian Sea Level from Coastal Observations in 1977–1991. – Atmospheric and Oceanic Physics, 2004, Vol. 40, No.6, pp. 737-746.
http://alexeylyubushin.narod.ru/Canonical_Coherences_of_Caspian_Sea_Level_Variations.pdf

Lyubushin A.A., P.H.A.J.M. van Gelder and M.V.Bolgov (2004) Spectral Analysis of Caspian Level Variations – Proceedings of OMAE 2004: 23rd International Conference Offshore Mechanics and Arctic Engineering, 20-25 June 2004, Vancouver, Canada.
http://proceedings.asmedigitalcollection.asme.org/proceeding.aspx?articleID=1630411
http://alexeylyubushin.narod.ru/Spectral_Analysis_of_Caspian_Sea_Level_Variations.pdf

Lyubushin A.A. and G.A. Sobolev (2006) Multifractal Measures of Synchronization of Microseismic Oscillations in a Minute Range of Periods – Izvestiya, Physics of the Solid Earth, vol.42, No.9, pp.734-744.
http://link.springer.com/article/10.1134%2FS1069351306090035
http://alexeylyubushin.narod.ru/Multifractal_Measures_of_Synchronization_of_Microseismic_Oscillations.pdf

Lyubushin A.A. (2008) Microseismic Noise in the Low Frequency Range (Periods of 1–300 min): Properties and Possible Prognostic Features – Izvestiya, Physics of the Solid Earth, Vol. 44, No. 4, pp. 275–290.
http://link.springer.com/article/10.1134/s11486-008-4002-6
http://alexeylyubushin.narod.ru/Low_Frequency_Microseismic_Noise_Properties_Possible_Prognostic_Features.pdf

Lyubushin A.A. (2009) Synchronization Trends and Rhythms of Multifractal Parameters of the Field of Low-Frequency Microseisms – Izvestiya, Physics of the Solid Earth, Vol. 45, No. 5, pp. 381–394.
http://link.springer.com/article/10.1134%2FS1069351309050024
http://alexeylyubushin.narod.ru/Trends_and_Rhythms_of_MicroSeisms_Synchronization.pdf

Lyubushin A. (2010) Multifractal Parameters of Low-Frequency Microseisms // V. de Rubeis et al. (eds.), Synchronization and Triggering: from Fracture to Earthquake Processes, GeoPlanet: Earth and Planetary Sciences 1, DOI 10.1007/978-3-642-12300-9_15, Springer-Verlag Berlin Heidelberg 2010, 388p., Chapter 15, pp.253-272.
http://link.springer.com/chapter/10.1007%2F978-3-642-12300-9_15
http://eqphys.narod.ru/olderfiles/1/MultiFract_SeismNoise.pdf

Lyubushin A.A., Klyashtorin L.B. (2012) Short term global *dT* prediction using (60-70)-years periodicity. Energy & Environment, vol. 23, No.1, pp.75-85.
http://multi-science.metapress.com/content/2w8r55204g511291/
http://alexeylyubushin.narod.ru/Global_dT_prediction_using_60-70_years_periodicity.pdf

Lyubushin A.A. (2014) Analysis of Coherence in Global Seismic Noise for 1997–2012. Izvestiya, Physics of the Solid Earth, Volume 50, No. 3, pp 325-333.
http://link.springer.com/article/10.1134%2FS1069351314030069
http://alexeylyubushin.narod.ru/Global_seismic_noise_coherence_ENG.pdf

Marple S.L.(Jr.) (1987) Digital spectral analysis with applications. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Sobolev G.A. and A.A. Lyubushin (2007) Microseismic Anomalies before the Sumatra Earthquake of December 26, 2004 – Izvestiya, Physics of the Solid Earth, vol.43, No.5, pp.341-353.
http://link.springer.com/article/10.1134%2FS1069351307050011
http://alexeylyubushin.narod.ru/Microseismic_Anomalies_before_Sumatra_Earthquake.pdf