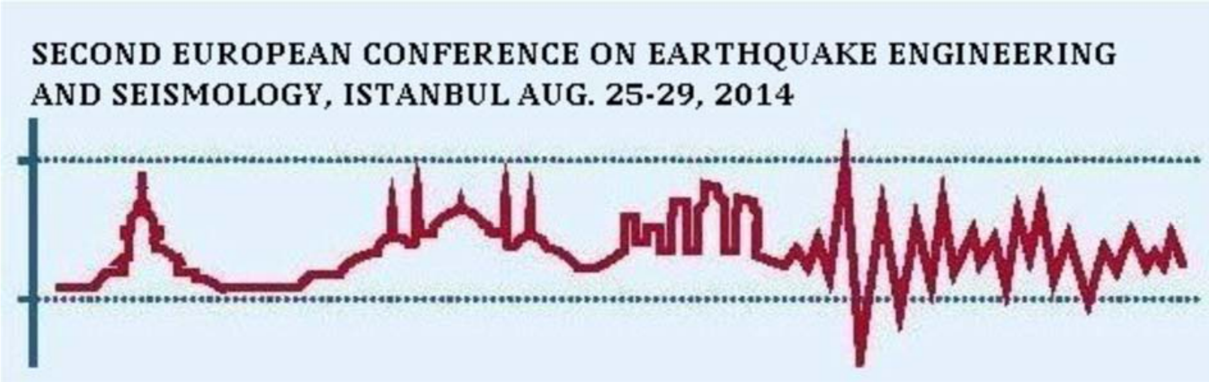

GLOBAL SEISMIC NOISE SYNCHRONIZATION AND SEISMIC DANGER

Alexey LYUBUSHIN[1]

ABSTRACT

The coherent behaviour of four parameters characterizing the global field of low-frequency (periods from 2 to 500 min) seismic noise is studied. These parameters include generalized Hurst exponent, multi-fractal singularity spectrum support width, normalized entropy of variance and kurtosis. The analysis is based on the data from 229 broadband stations of GSN, GEOSCOPE, and GEOFON networks for a 17 year period from the beginning of 1997 to the end of 2013. The entire set of stations is subdivided into eight groups, which, taken together, provide full coverage of the Earth. The daily median values of the studied noise parameters are calculated in each group. This procedure yields four 8-dimensional time series with a time step of 1 day with a length of 6209 samples in each scalar component. For each of the four 8-dimensional time series, the time-frequency diagram of the evolution of the spectral measure of coherence (based on canonical coherences) is constructed in the moving time window with a length of 365 days. Besides, for each parameter, the maximum frequency values are calculated as a measure of synchronization that depends on time only. Based on the conducted analysis, it is concluded that the increase in the intensity of the strongest ($M \geq 8.5$) earthquakes after the mega-earthquake on Sumatra on December 26, 2004 was preceded by the enhancement of synchronization between the parameters of global seismic noise over the entire time interval of observations since the beginning of 1997. This synchronization continues growing up to the end of the studied period (2013), which can be interpreted as a probable precursor of the further increase in the intensity of the strongest earthquakes all over the world.

INTRODUCTION

Study of the characteristics of noise in complex systems is one of the most promising directions of scientific research. This is a consequence of general trend in studying processes in complex nonlinear systems in physics, biology, finances and other fields where ambient noise is regarded as an important source of information. Such studies lie at the borderline of different disciplines since there is much more similarity in this field than the differences associated with the individual properties of the studied objects. In this sense, the study of such a complex system as the Earth constitutes no exception. The low-frequency seismic noise caused by the interaction between the lithosphere, atmosphere, and ocean has a complicated statistical structure, which contains the information about the preparation of the geological catastrophes including large earthquakes.

This paper is a continuation of series of papers Lyubushin (2008, 2009, 2010(a,b), 2011(a,b), 2012, 2013(a,b), 2014(a,b)) which were devoted to the analysis of different statistics obtained from seismic noise waveforms for the problems of earthquakes predictions.

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DATA

The seismic records were taken by requests to IRIS data base by the address http://www.iris.edu/forms/webrequest/ from 229 seismic stations of 3 global broadband seismic networks: Global Seismographic Network (http://www.iris.edu/mda/_GSN), GEOSCOPE (http://www.iris.edu/mda/G) and GEOFON (http://www.iris.edu/mda/GE).

Vertical components with sampling rate 1 Hz (LHZ-records) were downloaded for 17 years of observation since 01 Jan 1997 up to 31 Dec 2013. The initial LHZ-records were transformed to sampling time step 1 minute by calculating mean values within successive time intervals of the length 60 seconds. A further analysis is based on estimating statistical properties of low-frequency seismic noise waveforms (periods exceeding 2 minutes) within successive daily time intervals of the length 1440 samples with time step 1 minute.

Figure 1 presents positions of 229 broadband seismic stations all over the world and their splitting into 8 groups of stations. Each group has 3-letters identification code and the number of stations within each group is given in brackets. The names of the groups have the following abbreviation sense: the first letter is “N” or “S” what means North or South. The second letter is “E” or “W” what means East or West. Thus, initially all station were divided into 4 parts by splitting into North-East, North-West, South-East and South-West quarter-spheres. Finally each of 4 parts was split into North and South parts (the third letter is “N” or “S”) by the rule that the number of stations within each part must be approximately equal each other.

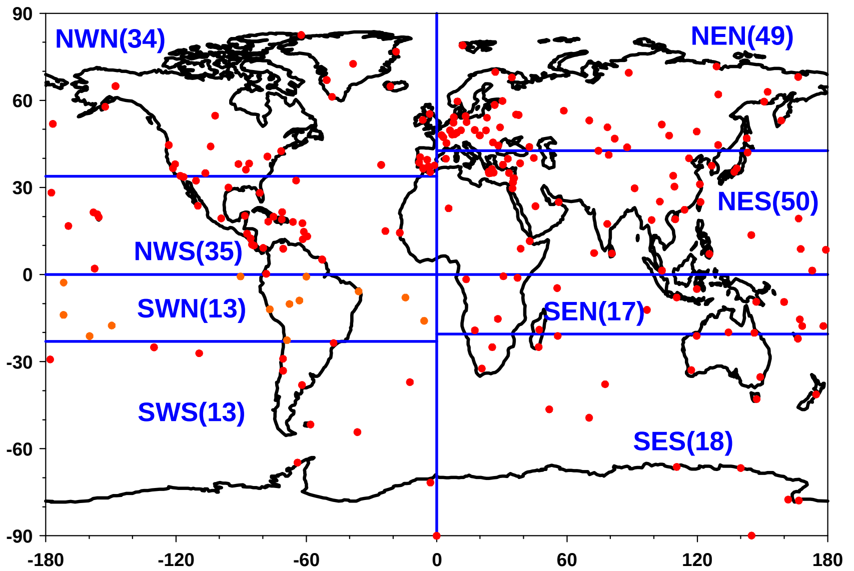

Figure 1. Positions of 229 broadband seismic stations and their splitting into 8 groups with number of stations in each group in brackets.

SEISMIC NOISE WAVEFORMS PARAMETERS

The seismic records from each station after coming to 1 minute sampling time step were split into adjacent time fragments of the length 1 day (1440 samples) and for each fragment 4 parameters of low-frequency daily seismic noise waveforms were calculated. Two of them are multifractal parameters: generalized Hurst exponent α^* and singularity spectrum support width $\Delta\alpha$. Two other seismic noise parameters are kurtosis κ and normalized entropy of variance $EntVar$. Thus, time series of α^*, $\Delta\alpha$, κ and $EntVar$ values with sampling time step 1 day were obtained from each of 229 seismic stations which are presented at the Fig.1. The Fig.2 illustrates the sequence of data transform operations.

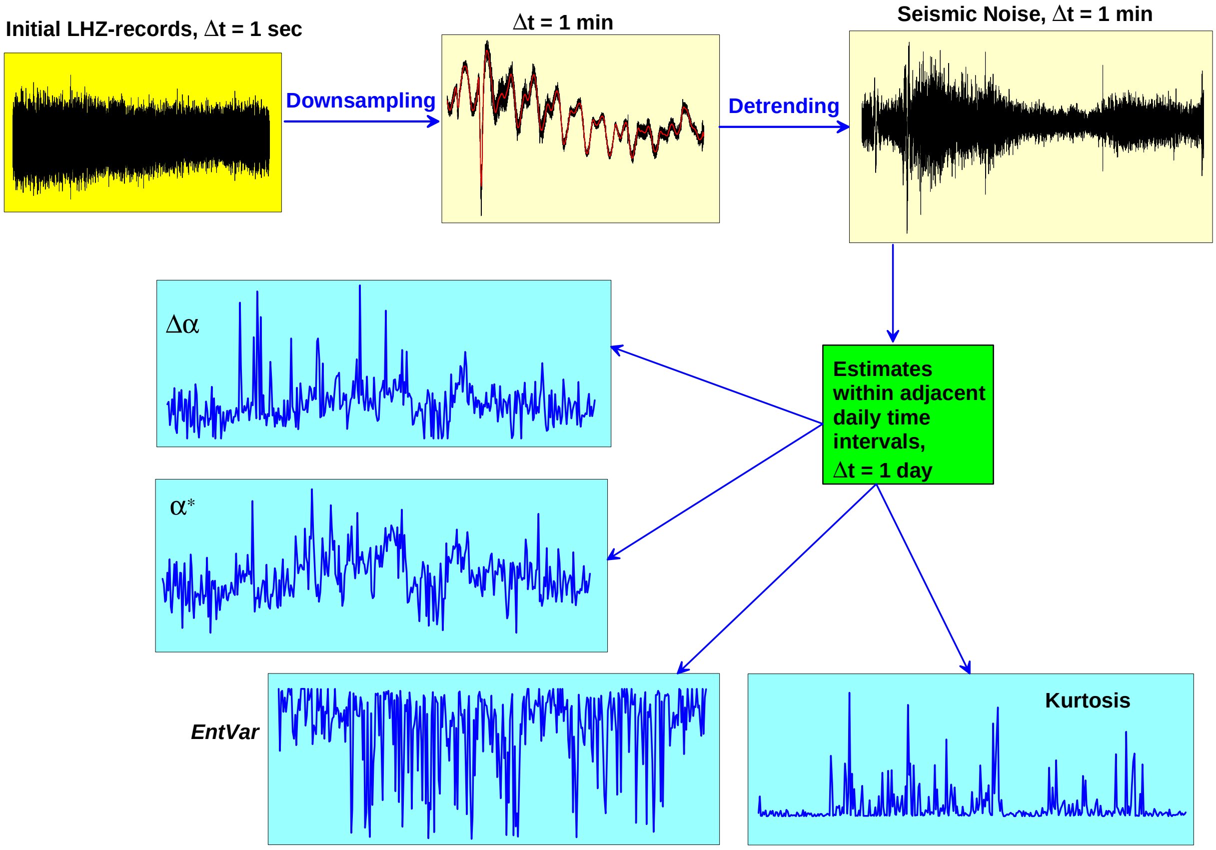

Fig.2. Scheme of data transform.

Estimates of multifractal properties α^* and $\Delta\alpha$ of low-frequency seismic noise were used in papers (Lyubushin, 2008, 2009, 2010(a,b), 2011(a,b), 2012, 2013(a,b), 2014(a,b)) for the purposes of earthquake prediction and dynamic estimate of seismic danger. The normalized entropy of seismic noise variance $EntVar$ was introduced in (Lyubushin, 2014(a)). A brief description of the used statistics is given below.

Multifractal singularity spectrum $F(\alpha)$ of the signal $X(t)$ is defined as a fractal dimensionality of time moments t_α which have the same value of local Lipschitz-Holder exponent $h(t) = \lim_{\delta \to 0} \dfrac{\ln(\mu(t,\delta))}{\ln(\delta)}$, i.e. $h(t_\alpha) = \alpha$, where $\mu(t,\delta) = \max_{t-\delta/2 \le s \le t+\delta/2} X(s) - \min_{t-\delta/2 \le s \le t+\delta/2} X(s)$ is a measure of signal variability in the vicinity of time moment t (Feder, 1988). If $X(t)$ is a usual self-similar monofractal signal with Hurst exponent value $0 < H < 1$ [Taqqu, 1988], then $F(H) = 1$, $F(\alpha) = 0 \, \forall \alpha \ne H$ but finite sample estimate of singularity spectrum does not obey these rigorous theoretical conditions of course.

Practically the most convenient method for estimating singularity spectrum is a multifractal detrended fluctuation analysis (DFA) (Kantelhardt et al., 2002) which is used here. The function $F(\alpha)$ could be characterized by following parameters: $\alpha_{\min}, \alpha_{\max}, \Delta\alpha = \alpha_{\max} - \alpha_{\min}$ and α^* - an argument providing maximum to singularity spectra: $F(\alpha^*) = \max_\alpha F(\alpha)$. Parameter α^* is called a generalized Hurst exponent and it gives the most typical value of Lipschitz-Holder exponent. Parameter $\Delta\alpha$, singularity spectrum support width, could be regarded as a measure of variety of stochastic behavior. It should be noticed that usually $F(\alpha^*) = 1$ – maximum of singularity spectra equals to the dimensionality of embedding set, i.e. to dimensionality of time interval. For removing scale-dependent trends (which are mostly caused by tidal variations) in multifractal DFA-method of singularity spectrums estimates a local polynomials of the 8-th order were used.

Kurtosis κ is defined by the formula (Cramer, 1999):

$$\kappa = \left\langle (\Delta x)^4 \right\rangle \Big/ \left\langle (\Delta x)^2 \right\rangle^2 - 3 \qquad (1)$$

Here Δx is deflection of the daily noise waveform from trend which is chosen as polynomial of the 8^{th} order, $\langle ... \rangle$ is the symbol of sample estimate of mean value. Kurtosis characterizes the sharpness of probability distribution form and gives a measure of deflection of Δx from normal distribution for which $\kappa = 0$. If $\kappa \gg 1$ then signal is called leptokurtotic and this property means the existence of "fatter tails" of distribution. The seismic noise is leptokurtotic.

Let us introduce normalized entropy of variance (Lyubushin, 2014(a)). For this purpose let us split each daily noise waveform into 24 parts which correspond to adjacent hour intervals. Let $V_\alpha, \alpha = 1, ..., m_V = 24$ be variance values which are calculated for increments of waveforms (after removing tidal trends from daily waveforms) within adjacent time intervals of the length 60 samples, i.e. for adjacent hour time intervals. After that let us calculate normalized entropy of variance distribution by the formula:

$$EntVar = -\sum_{\alpha=1}^{m_V} p_\alpha \cdot \log(p_\alpha) / \log(m_V), \quad p_\alpha = V_\alpha / \sum_{\beta=1}^{m_V} V_\beta, \quad 0 \le EntVar \le 1 \qquad (2)$$

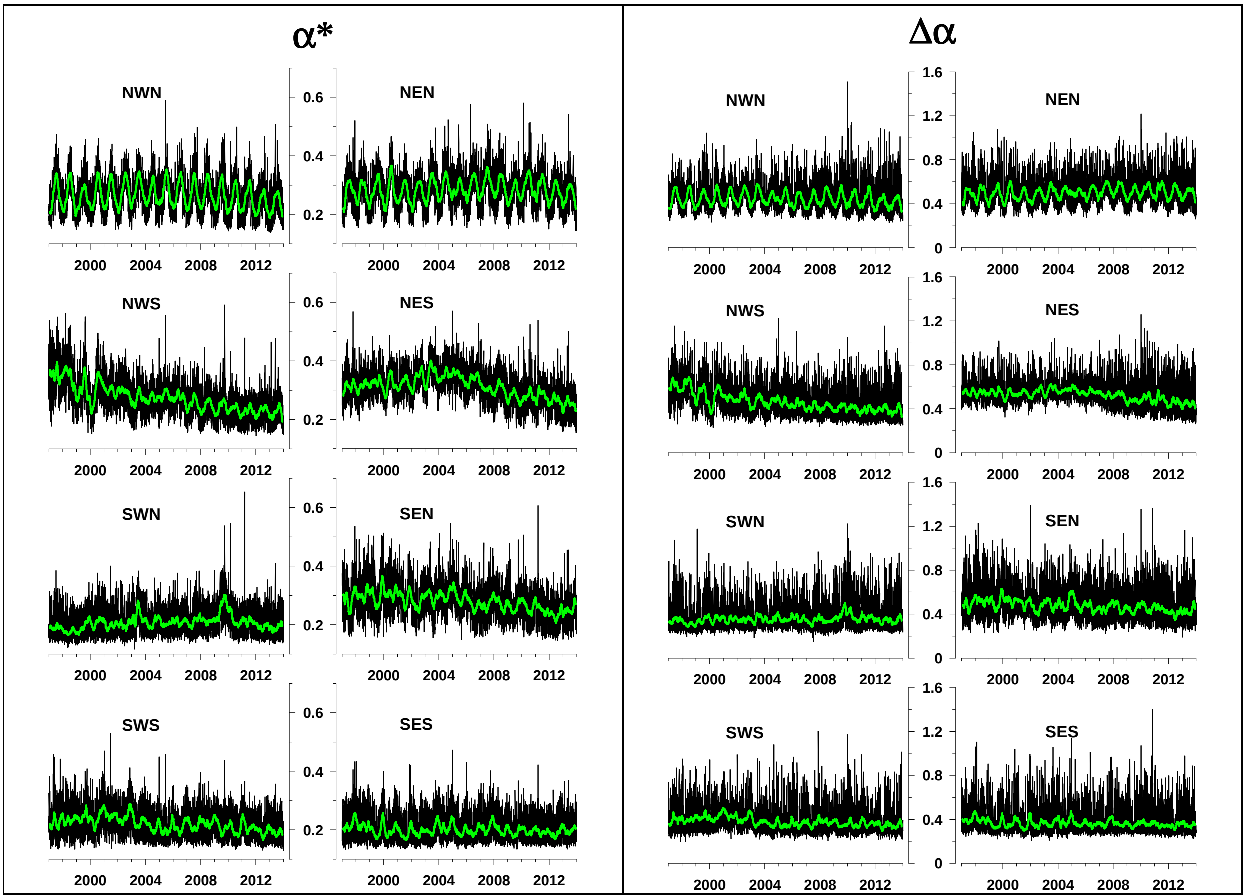

Fig.3. Daily median values of 2 multi-fractal parameters of low-frequency seismic noise from 8 groups of stations presented at Fig.1. Bold green lines are graphics of running average within moving time window of the length 57 days.

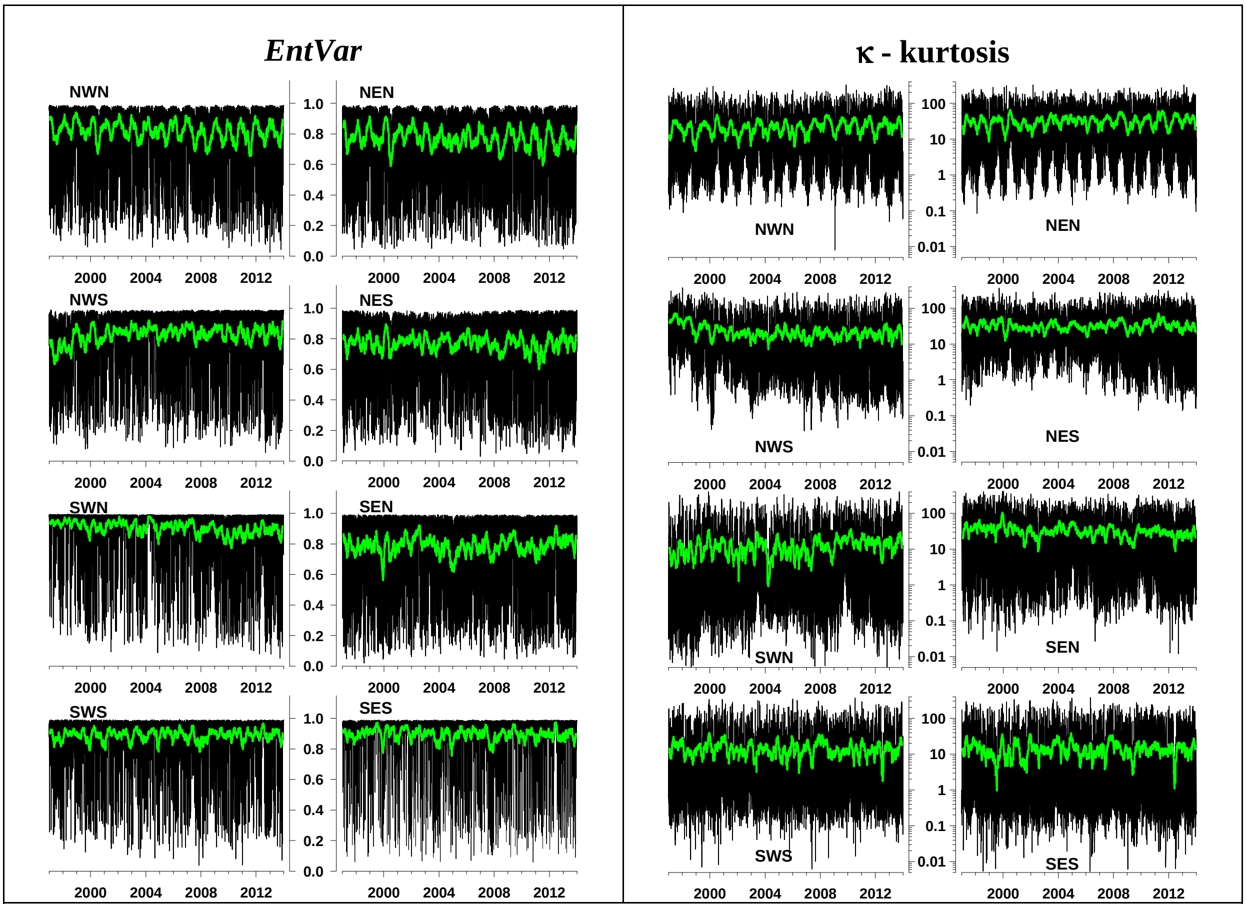

Fig.4. Daily median values of normalized entropy of variance and kurtosis of low-frequency seismic noise from 8 groups of stations presented at Fig.1. Bold green lines are graphics of running average within moving time window of the length 57 days.

Figures 3 and 4 presents 16 graphics of median values of 4 statistics α^*, $\Delta\alpha$, κ and $EntVar$ calculated each day of 17 years (1997-2013, 6209 daily samples in each scalar time series) from 8 groups of stations which are presented at the Fig.1. It is interesting to notice that variations of all noise parameters for 2 groups NWN and NEN have the most explicit annual periodic components. The next step consists in estimating of evolution of multiple coherence measure for variations of 8-dimensional time series of each seismic noise properties in a moving time window.

MULTIPLE SPECTRAL COHERENCE MEASURE

Multiple spectral coherence measure was suggested in (Lyubushin, 1998) for multidimensional time series processing in the problems of geophysical monitoring. In the papers (Lyubushin, 1999; Lyubushin et al., 2003, 2004; Lyubushin, Sobolev, 2006; Sobolev, Lyubushin, 2007; Lyubushin, 2009, 2010(b); Lyubushin, Klyashtorin, 2012; Lyubushin, 2014(b)) this spectral measure was applied to different problems of multidimensional time series analysis in geophysics, meteorology, hydrology and climate sciences.

Canonical coherences are the generalization of usual squared coherence spectrum between two scalar time series for the case when two vector time series are considered: m-dimensional time series $X(t)$ and n-dimensional time series $Y(t)$. Here t is an integer time index. Without loss of generality let us suppose that $n \le m$. Squared maximum canonical coherence $\rho_1^2(\omega)$ between multiple time series $X(t)$ and $Y(t)$ is computed as maximum eigenvalue of the following frequency-dependent matrix (Brillinger, 1975; Hanan, 1970):

$$U(\omega) = S_{xx}^{-1/2} S_{xy} S_{yy}^{-1} S_{yx} S_{xx}^{-1/2} \quad (3)$$

Here ω is the frequency, $S_{xx}(\omega)$ is spectral matrix of the size $m \times m$ of time series $X(t)$, $S_{xy}(\omega)$ is cross-spectrum matrix of the size $m \times n$ between time series $X(t)$ and $Y(t)$, $S_{yx}(\omega) = S_{xy}^{H}(\omega)$, "$H$" is the sign of Hermitian conjunctions, $S_{yy}(\omega)$ is spectral matrix of the size $n \times n$ of time series $Y(t)$. The value of $\rho_1^2(\omega)$ is used instead of usual squared coherence spectrum when 2 scalar time series are regarded, i.e. when $m = n = 1$.

Let us introduce the notion of by-component canonical coherence $\nu_i^2(\omega)$ of q-dimensional time series $Z(t)$ as squared maximum canonical coherences when in the formula (3) $Y(t)$ is the scalar i-th component of q-dimensional time series $Z(t)$ whereas $X(t)$ is $(q-1)$-dimensional time series composed of all other scalar components of $Z(t)$. Thus, in the formula (3) $n = 1$, $m = (q-1)$. In our case $q = 8$.

The value $\nu_i^2(\omega)$ is the measure of connection of variations of the i-th component of q-dimensional time series $Z(t)$ with variations of all other scalar components of $Z(t)$ at the frequency ω. The inequality $0 \le |\nu_i(\omega)| \le 1$ is fulfilled, and the closer the value of $|\nu_i(\omega)|$ to unity, the stronger the linear relation of variations at the frequency ω of the i-th scalar series to analogous variations in all other series. Now we can define the multiple spectral coherence measure by formula:

$$\lambda(\omega) = \prod_{i=1}^{q} |\nu_i(\omega)| \quad (4)$$

The value (4) provides a frequency-dependent measure of linear joint synchronization of variations of all scalar components of time series $Z(t)$ at the frequency ω. For calculating the measure (4) it is necessary to estimate spectral matrix $S_{zz}(\omega)$ of $Z(t)$ of the size $q \times q$. For this purpose we use vector autoregression model (Marple, Jr., 1987):

$$Z(t) + \sum_{k=1}^{p} A_k \cdot Z(t-k) = e(t) \quad (5)$$

where t is p an autoregression order, A_k are matrices of autoregression coefficients of the size $q \times q$, $e(t)$ is q-dimensional residual signal with zero mean and covariance matrix $\Phi = M\{e(t)e^T(t)\}$ of the size $q \times q$. Matrices A_k and Φ are defined using Durbin-Levinson procedure and the spectral matrix is calculated using formula:

$$S(\omega) = F^{-1}(\omega) \cdot \Phi \cdot F^{-H}(\omega), \quad F(\omega) = E + \sum_{k=1}^{p} A_k \cdot \exp(-i\omega k) \quad (6)$$

where E is a unit matrix of the size $q \times q$.

Let us consider moving time window of the certain length and let τ be the time coordinate of right-hand end of moving time window. If the function (4) will be estimated within each time window independently then we will have time-frequency function:

$$\lambda(\tau, \omega) = \prod_{i=1}^{q} |\nu_i(\tau, \omega)| \quad (7)$$

The value (7) presents the evolution of linear synchronization measure for multiple time series $Z(t)$. Usually some preliminary operations are fulfilled within each time window before estimating spectral matrix (6), such as removing linear trend and coming to increments. We use autoregression order $p = 5$ within moving time window of the length 365 days taken with mutual shift 7 days.

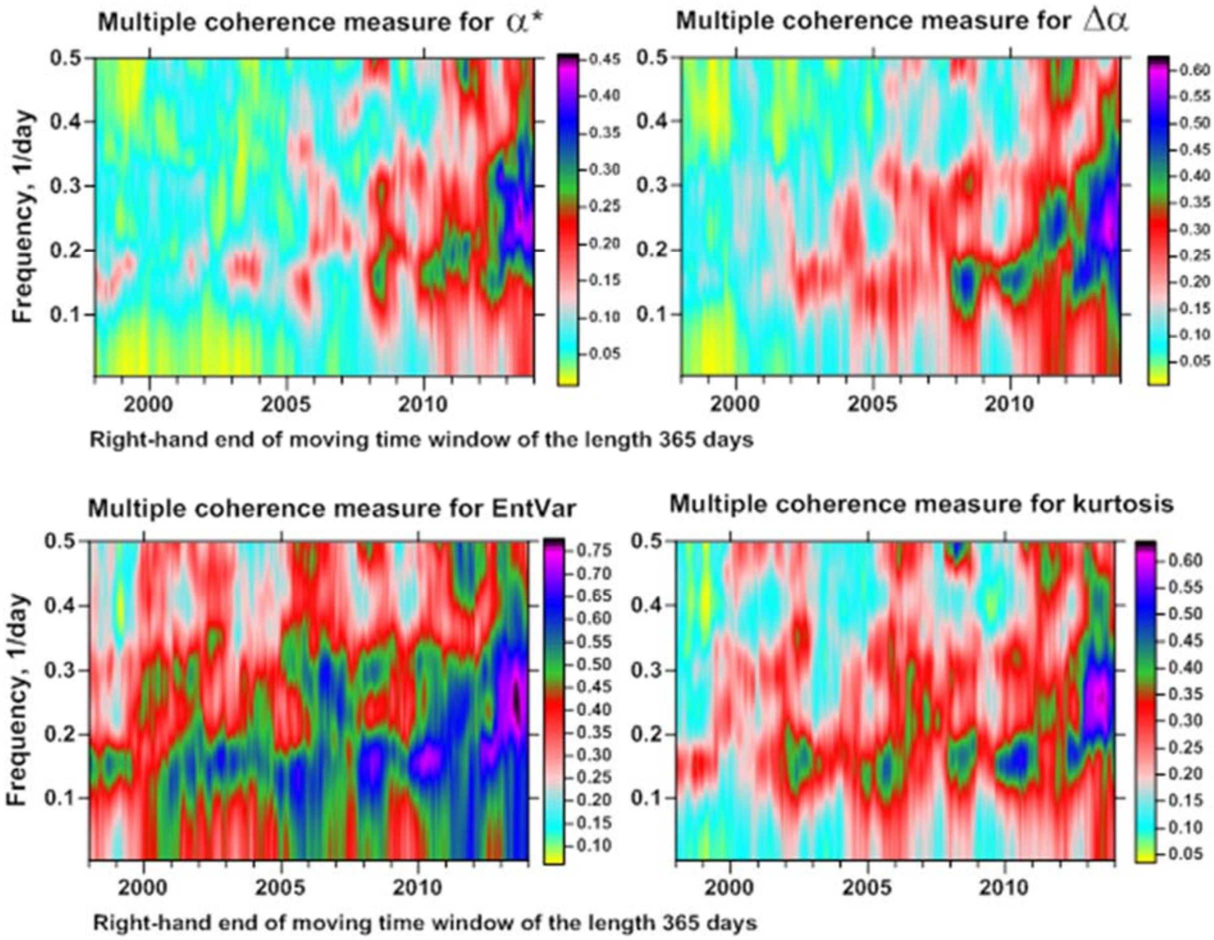

Figure 5. Time-frequency diagrams of evolution of multiple spectral coherence measure (7) for four 8-dimensional time series of daily seismic noise properties which are presented at figures 3 and 4 within moving time window of the length 365 days with mutual shift 7 days.

Figure 5 shows the time-frequency diagrams of the spectral measure of coherence (7) for the variations of 4 studied parameters of the seismic noise (Figures 3 and 4). It is evident that they are characterized by bursts of coherence with increasing amplitude, the maxima occur in bursts of coherence periods range from 5 to 10 days.

Besides time-frequency diagram (7) let us consider pure temporal measure of coherence which is defined for each value of τ as maximum value of (7) with respect to frequency:

$$\lambda_{\max}(\tau) = \max_{\omega} \lambda(\tau, \omega) \tag{8}$$

Note that the quantity (8) is an analogue of the multiple correlation coefficient, calculated in a moving time window. Due to the fact that the maximum in (8) is taken over all frequencies, this factor takes into account the time shifts between the scalar components of multivariate time series within the current time window.

Thus, we obtain 4 maximum coherence measures as a function of the time position of the right-hand end of the moving time window whose graphs are presented in Figure 6. We see that general trends (bold blue lines present polynomial trends of the 3rd order) of increasing coherence between variations of different parameters of seismic noise taken from 8 groups of seismic stations covering all surface of the Earth are clearly noticed.

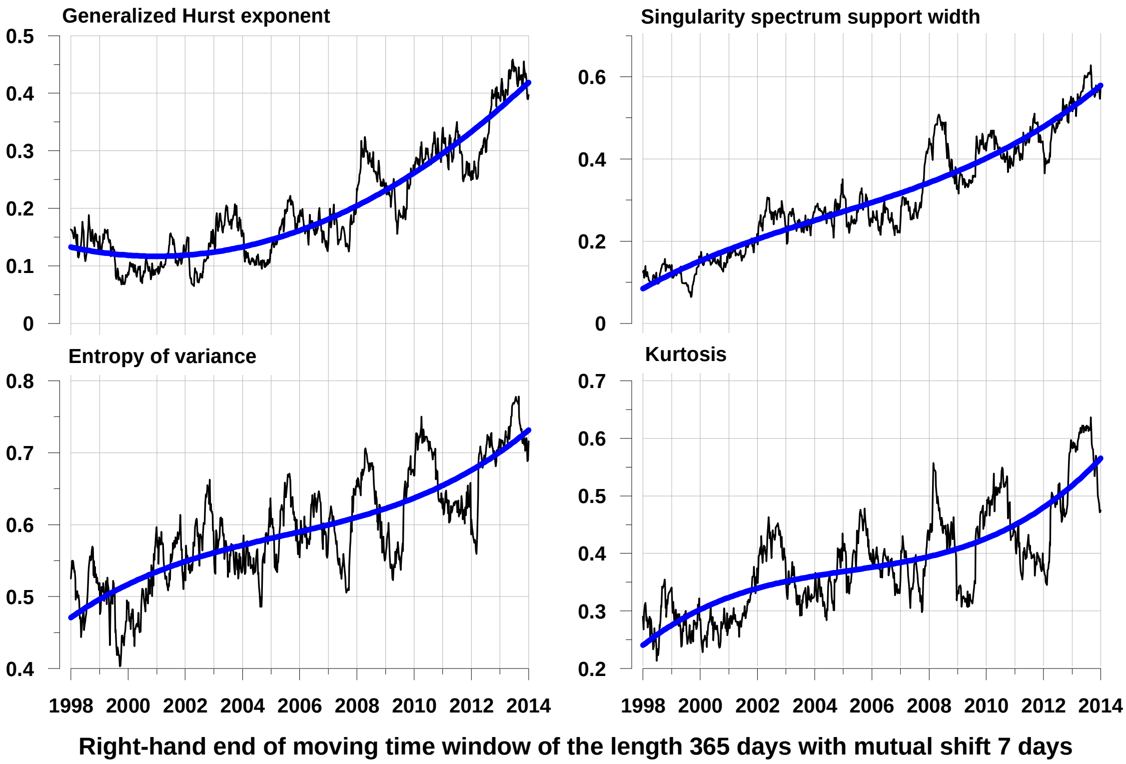

Figure 6. Maximum (with respect to frequency) values (8) of coherence measures (7) for 4 properties of low-frequency seismic noise corresponding to time-frequency diagrams at figure 5. Bold blue lines present polynomial trend of the 3rd order.

DISCUSSION AND CONCLUSION

In this paper the analysis covers the data from a large number of broadband seismic stations globally distributed all over the world with the aim to identify the variability of global effects of synchronization in seismic noise in a moving time window. It is known that, starting from the mega-earthquake on Sumatra on December 26, 2004, the Earth experienced a series of strongest earthquakes ($M \geq 8.5$), which have not occurred since the beginning of 1965. This information is presented in the Table 1. We can notice that among these 17 strongest seismic events 6 took place during the last 10 years. During previous time interval of 40 year duration, 1965-2004, no strongest events took place at all. Moreover, among these 6 strongest earthquakes 4 occurred during last 7 years, since 2007. Thus, the last ten years are marked by the significant increasing of seismic intensity with acceleration.

Table 1. Strongest earthquakes, $M \geq 8.5$, from the beginning of 20th century
Source: http://earthquake.usgs.gov/earthquakes/world/10_largest_world.php

Date	Magnitude	Latitude	Longitude	Date	Magnitude	Latitude	Longitude
1906.01.31	8.8	1	-81.5	1964.03.28	9.2	61.02	-147.65
1922.11.11	8.5	-28.55	-70.5	1965.02.04	8.7	51.21	178.5
1923.02.03	8.5	54	161	2004.12.26	9.1	3.3	95.78
1938.02.01	8.5	-5.05	131.62	2005.03.28	8.6	2.08	97.01
1950.08.15	8.6	28.5	96.5	2007.09.12	8.5	-4.438	101.367
1952.11.04	9.0	52.76	160.06	2010.02.27	8.8	-35.846	-72.719
1957.03.09	8.6	51.56	-175.39	2011.03.11	9.0	38.322	142.369
1960.05.22	9.5	-38.29	-73.05	2012.04.11	8.6	2.311	93.063
1963.10.13	8.5	44.9	149.6				

The following questions now arise: how is this activation reflected in the coherence of time series of the parameters characterizing the global seismic noise? From graphics of evolution of multiple coherences at the figures 5 and .6 it is evident that an increasing of synchronization is observed for all properties of seismic noise. In the theory of complex systems a phenomenon of increasing radius of correlations of statistical fluctuations (i.e. ambient noise of the system), "critical opalescence" in the theory of phase transitions, is a well-known indicator of approaching to abrupt changes of the system, to catastrophe (Gilmore, 1981; Nicolis, Prigogine, 1989). Thus, a dramatic increasing of strongest earthquakes rate is observed starting from Sumatra mega-earthquake at 26 Dec of 2004, especially starting from 2007 and this increasing corresponds to increasing trend of multiple coherence measures of global seismic noise at the Fig.5-6. On this foundation a hypothesis that the rate of strongest earthquakes could increase in the nearest future could be formulated.

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