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Research on Seismic Regime Using Linear Model of Intensity of Interacting Point Processes

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Introduction

The article examines the problem of a quantitative description of the influence of the seismic regime in one region on the seismic regime in another region. Observations of long-term earthquake precursors give evidence of the presence of such a correlation even if the considered regions are separated from one another by considerable distances (up to several hundred kilometers). These facts fully conform to the plate tectonics concept and the capability for rapid transmission of changes in crustal stresses by rigid plates. Mathematically the problem is reduced to formalization of a model of the influence of one or several point processes on another. The traditionally employed approaches of cross-covariation and cross-spectral analysis require preliminary transformation of point processes to continuous time series with a constant time quantization interval. However, if the initial point processes contain considerable time intervals without events and relatively short time intervals of clustering of events (as occurs precisely for a seismic process), due to their transformation to continuous time series using simple summation in a time window the result is essentially non-Gaussian processes containing strong amplitude surges, which makes difficult application of covariation analysis and reduces time resolution.

Another approach to the description of the interaction of point processes involves the conservation of their "point nature" and the construction of parametric models of process intensity [*Koks and L'yuis,* 1969] which would contain parts corresponding to both the influence of the process in itself (autoregression or self-exciting part) and the influence of other point processes on the considered process (regression part, external excitation). Precisely such an approach was adopted in [*Ogata and Akaike,* 1982; *Ogata et al.,* 1982] and it was used in studying the reciprocal influence of the seismic regime in two active regions in Japan [*Ogata et al.,* 1982].

Here we will use generalization to a multidimensional case of a simple variant of the class of intensity models with linear parameters proposed in [*Ogata and Akaike,* 1982; *Ogata et al.,* 1982] and it will be used in studying the characteristics of the seismic regime in California during 1963 – 1990 from the point of view of the reciprocal influence of different parts of this region.

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Model of Interaction of Processes

We will divide the considered seismically active region into m nonintersecting regions. We will examine the point processes $\{t_j^{(\alpha)}, M_j^{(\alpha)}, j = 1, \ldots, N_\alpha\}$, $\alpha = 1, \ldots, m$, where $t_j^{(\alpha)}$ are the earthquake times in the α–th region, $M_j^{(\alpha)}$ are event magnitudes.

Assume that $\lambda^{(\beta)}(t)$ is nominal process intensity (mean number of events per unit time) in the β–th region with a fixed past up to the time t. We represent it in the following form:

$$\lambda^{(\beta)}(t) = \mu^{(\beta)} + \sum_{\alpha=1}^{m} b_\alpha^{(\beta)} g_\alpha(t) \quad (1)$$

where $\mu^{(\beta)}$, $b_\alpha^{(\beta)}$ are non-negative parameters; the g_α functions have the form:

$$g_\alpha(t) = \sum_{t_j^{(\alpha)} < t} \exp(-(t - t_j^{(\alpha)})/\tau + r(M_j^{(\alpha)} - M_0)) \quad (2)$$

Thus, each event "generates" a time influence function having the form of a diminishing exponent with the characteristic time τ, multiplied by the coefficient $\exp(r(M_j^{(\alpha)} - M_0))$, increasing exponentially with an increase in event magnitude. Here τ and r, $r \geq 0$ are model parameters, M_0 is the minimum magnitude of the events taken into consideration. The sum of all such exponentially diminishing time response functions for events rigorously less than the current time value t forms the general influence function $g_\alpha(t)$ of the α–th region. The $b_\alpha^{(\beta)}$ parameters are scaling factors and their values, in essence, determine the degree of influence of the α–th region on the β–th region. The $b_\beta^{(\beta)}$ values determine the degree of influence of the region itself (self-exciting intensity component). With respect to the $\mu^{(\beta)}$ parameter, its value reflects the contribution of the purely random Poisson component to process intensity. The greater the $\mu^{(\beta)}$ value, the more "random" is the regime of the β–th region.

We note that the conclusions drawn concerning the relative contribution of one component or another (self-exciting, external or stochastic) to intensity (1) are dependent on the values of the time constant τ and the parameter r with which the values of the $(\mu,\ b_\alpha)$ parameters are determined. Formally it also would be possible to include $(\tau,\ r)$ in the list of parameters to be determined. However, in such cases almost always a situation arises with the tending of τ to zero when evaluating the parameters by the maximum likelihood method [*Ogata and Akaike,* 1982; *Ogata et al.,* 1982]. In addition, for each region in such cases its own values of the $(\tau,\ r)$ parameters would be obtained, which makes it difficult to compare the results obtained for different β. Finally, the inclusion of the $(\tau,\ r)$ parameters in the list of model parameters subject to identification would result in an increase in calculation time by one-two orders of magnitude because then it would be necessary once again to rescale the values of the response functions for each new $(\tau,\ r)$ pair. Accordingly, the values of the $(\tau,\ r)$ parameters also are registered and the parameters of the model (1) are determined for each specific $(\tau,\ r)$ pair. One τ value or another is interpreted as the time scale on which we estimate the relative contribution of the components to intensity.

The logarithmic function of nominal probability (partial likelihood) for a nonstationary point process for the β-th region is written in the following form [*Koks and L'yuis,* 1969; *Ogata and Akaike,* 1982; *Ogata et al.,* 1982; *Cox,* 1975]:

$$\ln(L_\beta) = \sum_{j=1}^{N_\beta} \ln(\lambda^{(\beta)}(t_j^{(\beta)})) - \int_0^T \lambda^{(\beta)}(s)ds \qquad (3)$$

where $[0,T]$ is the observation interval. It can be shown [*Cox,* 1975] that with adequately general conditions of regularity of a multidimensional point process in the considered regions the estimates of the $(\mu,\ b_\alpha)$ parameters maximizing (3) when $T \longrightarrow \infty$ have the very same justifiability and asymptotic efficiency properties as the standard maximum likelihood evaluations.

Thus, it is necessary to find the maximum of the function (3) from the $(\mu,\ b_\alpha)$ parameters with fixed τ. It is easy to check that the following relation applies

$$\mu\frac{\partial \ln(L_\beta)}{\partial\mu} + \sum_{\alpha=1}^{m} b_\alpha^{(\beta)}\frac{\partial \ln(L_\beta)}{\partial b_\alpha^{(\beta)}} = N_\beta - \int_0^T \lambda^{(\beta)}(s)ds \quad (4)$$

Due to the non-negative character of the $(\mu,\ b_\alpha)$ parameters each of the terms on the left-hand side of (4) becomes equal to zero at the point of the maximum of the function (3) because if the parameter is positive the corresponding partial derivative becomes equal to zero, but if the maximum is attained at the boundary, the value of the parameter becomes equal to zero. Accordingly, the following condition is satisfied at the point of the maximum of the likelihood function:

$$\int_0^T \lambda^{(\beta)}(s)ds = N_\beta \qquad (5)$$

Condition (5) represents a rather natural normalization condition. We substitute expression (1) into (5) and divide by the length of the total observation interval T. Then in place of (5) we obtain:

$$\mu^{(\beta)} + \sum_{\alpha=1}^{m} b_\alpha^{(\beta)}\bar{g}_\alpha = \mu_0^{(\beta)} \qquad (6)$$

where $\bar{g}_\alpha$ is the mean value of the $g_\alpha(t)$ function for the observation interval and $\mu_0^{(\beta)} = N_\beta/T$ is the mean intensity of the β-th region (coinciding with the evaluation of the intensity parameter of a purely Poisson process).

In equation (6) we divide both sides by $\mu_0^{(\beta)}$ and introduce the quantities:

$$k_\mu^{(\beta)} = \mu_\beta/\mu_0^{(\beta)},\ k_\alpha^{(\beta)} = b_\alpha^{(\beta)}\bar{g}_\alpha/\mu_0^{(\beta)} \qquad (7)$$

These quantities (obviously they can vary from 0 to1) can be called the relative contributions to the mean intensity of the β-th region: $k_\mu^{(\beta)}$ is the contribution of the stochastic (Poisson) component to the mean intensity, $k_\alpha^{(\beta)}$ is the contribution of the α-th region to the mean intensity of the β-th region (coefficient of the influence of α on β). The diagonal terms of the $m \times m$ matrix $k_\alpha^{(\beta)} - k_\beta^{(\beta)}$ are the influence coefficients of the β-th region in itself (coefficients of self-excitation of the β-th region). It follows from (6) that

$$k_\mu^{(\beta)} + \sum_{\alpha=1}^{m} k_\alpha^{(\beta)} = 1 \qquad (8)$$

Thus, our analysis will involve construction and interpretation of the influence matrices $(k_\mu^{(\beta)}, k_\alpha^{(\beta)})$, $\alpha = 1,\dots,m$; $\beta = 1,\dots,m$ for different (τ, r) values.

By making simple computations it can be shown that the matrix of second derivatives of function (3) for the $(m\ +\ 1)$-dimensional vector of parameters $c = (\mu, b_\alpha), \alpha = 1,\dots,m$ is equal to

$$\frac{\partial^2 \ln(L_\beta)}{\partial c_i \partial c_j} = -\sum_{k=1}^{N_\beta} \frac{f_i(t_k^{(\beta)})f_i(t_k^{(\beta)})}{\left(\sum\limits_{p=0}^{m} c_p f_p(t_k^{(\beta)})\right)^2} \qquad (9)$$

$$i,j = 0,1,\dots,m$$

where $c_0 \equiv \mu, c_\alpha \equiv b_\alpha; f_0(t) \equiv 1, f_\alpha(t) \equiv g_\alpha(t)$, $\alpha = 1,\dots,m$. It follows from formula (9) that the Hessian of function (3) is negatively determined and therefore the vector of parameters $c = (\mu, b)$ with non-negative components maximizing the function (3) is unique.

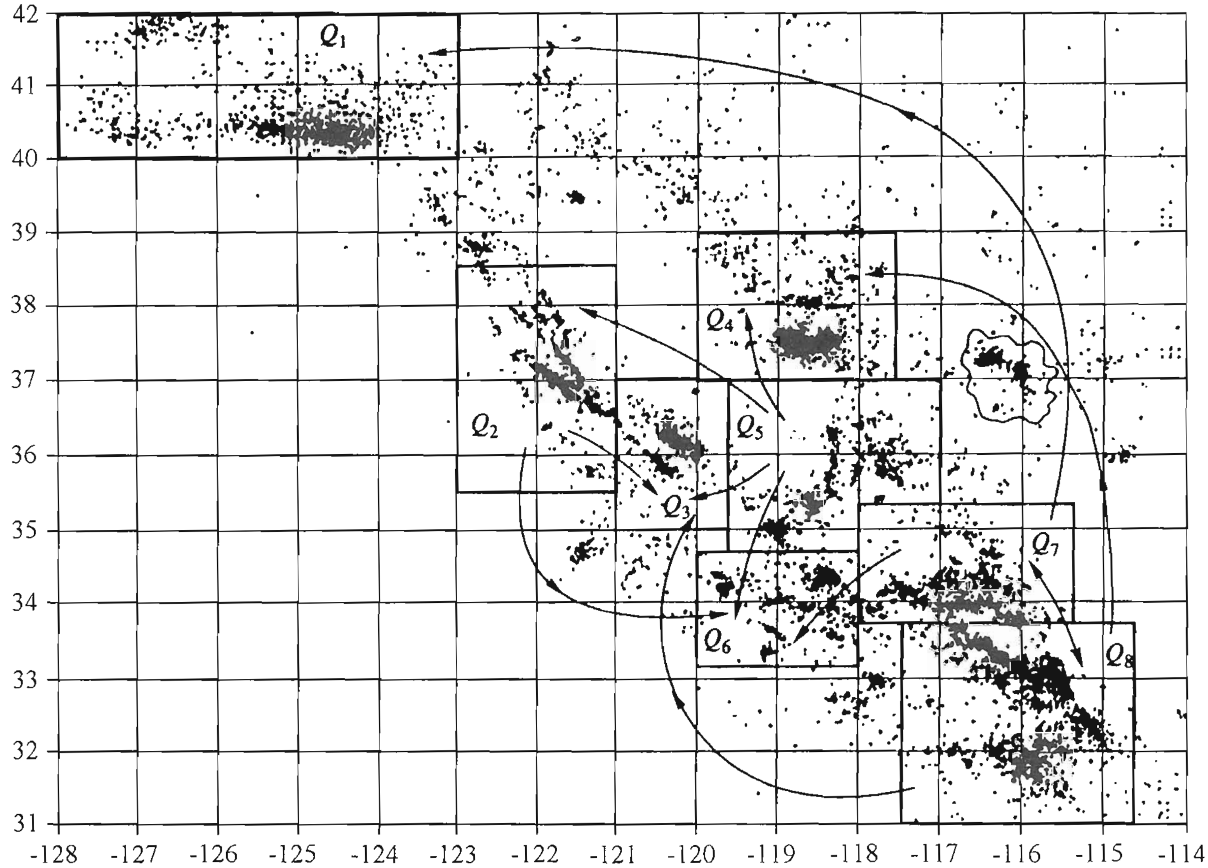

Figure 1.

Formula (9) can be used for approximately determining the dispersion of evaluations of the vector of parameters $\vec{\kappa} = (k_\mu^{(\beta)}, k_\alpha^{(\beta)}), \alpha = 1, \ldots, m$. Since the $\vec{\kappa}$ vector is related to the vector c by linear relations (7), by computing the matrix (9) and using (7) it is easy to obtain the matrix of second derivatives of the function (3) for the $\vec{\kappa}$ vector. Taken with a minus sign, this matrix is approximately equal to the Fisher matrix for evaluating the $\vec{\kappa}$ vector, but if it is inverted we obtain an approximate matrix for the covariation of the errors of the components of the $\vec{\kappa}$ vector. Thus, the diagonal elements of the inverted matrix are the approximate values of the quadratic dispersion of the evaluations of the parameters $(k_\mu^{(\beta)}, k_\alpha^{(\beta)})$.

For solving the problem of maximizing the function (3) use was made of the gradient method [*Pchenichnyy and Danilin,* 1975] with projection onto the limitations $\mu \geq 0$, $b_\alpha \geq 0$ and onto the hyperplane (6) to which, from the necessary conditions of the extremum, the sought-for vector of parameters should belong.

Processing Results

A catalogue of earthquakes of the Western United States, Southern Canada and Northern Mexico, covering the time period from 1963 to 1990 and consisting of 16 815 events (including explosions at the Nevada test site, which, naturally, were excluded from the analysis) [*Global...,* 1989-1990] was used in the processing. The minimum magnitude of the events included in the catalogue was 3, but weak events are not representative in all parts of the region covered by the catalogue: in peripheral regions there are disproportionately few weak events (low density of the seismic network); only for magnitudes beginning with 4.5 is the representativeness uniform for the entire region. With respect to the central part (western coast of the United States, California, about 13000 events), there all the magnitudes present in the catalogue ($M_{\min} = 3, M_{\max} = 7.2$) are representative. The figure shows the distribution of epicenters of all the events for this central part; the region of the

Table 1a. Initial subcatalogues, $r = 0, \tau = 100$ days

	0	1	2	3	4	5	6	7	8
1	.357	.600	.000	.000	.043	.000	.000	.000	.000
2	.201	.000	.799	.000	.000	.000	.000	.000	.000
3	.051	.000	.006	.816	.003	.070	.000	.000	.053
4	.056	.000	.000	.000	.942	.000	.002	.000	.000
5	.463	.000	.000	.001	.000	.529	.007	.000	.000
6	.065	.075	.000	.000	.000	.000	.861	.000	.000
7	.282	.000	.062	.000	.000	.000	.000	.657	.000
8	.191	.000	.000	.000	.000	.000	.000	.000	.809

Table 1b. Principal tremors, $r = 0, \tau = 100$ days

	0	1	2	3	4	5	6	7	8
1	.098	.801	.000	.000	.000	.000	.000	.101	.000
2	.226	.000	.541	.000	.000	.233	.000	.000	.000
3	.000	.000	.215	.147	.000	.223	.000	.000	.414
4	.000	.000	.000	.045	.599	.182	.017	.003	.153
5	.671	.000	.000	.017	.040	.253	.000	.019	.000
6	.053	.025	.224	.000	.000	.255	.311	.132	.000
7	.000	.013	.079	.000	.000	.008	.082	.413	.405
8	.111	.000	.000	.000	.000	.011	.033	.177	.668

Table 2a. Initial subcatalogues, $M_0 = 3.5, r = 0.5, \tau =$ 100 days

	0	1	2	3	4	5	6	7	8
1	.496	.471	.000	.000	.033	.000	.000	.000	.000
2	.254	.000	.746	.000	.000	.000	.000	.000	.000
3	.132	.000	.000	.819	.005	.011	.000	.000	.033
4	.071	.000	.000	.000	.928	.000	.001	.000	.000
5	.469	.000	.000	.000	.000	.521	.010	.000	.000
6	.039	.110	.000	.000	.000	.000	.851	.000	.000
7	.360	.000	.027	.000	.000	.000	.000	.613	.000
8	.279	.000	.000	.000	.000	.000	.000	.000	.721

Table 2b. Principal tremors, $M_0 = 3.5, r = 0.5, \tau =$ 100 days

	0	1	2	3	4	5	6	7	8
1	.270	.709	.000	.000	.000	.000	.000	.000	.021
2	.583	.000	.417	.000	.000	.000	.000	.000	.000
3	.272	.000	.106	.154	.000	.122	.000	.000	.347
4	.322	.000	.000	.000	.591	.000	.000	.000	.087
5	.687	.000	.000	.000	.011	.281	.000	.021	.000
6	.105	.000	.182	.000	.000	.230	.319	.117	.048
7	.186	.000	.030	.000	.043	.000	.000	.471	.271
8	.332	.000	.000	.000	.000	.000	.000	.000	.668

Nevada test site is surrounded by a wavy line.

In the considered region eight subregions are defined which take in quite dense seismicity spots and bear the conditional designations $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7$ and Q_8 (see Figure 1).

The computations were made for events having a magnitude greater than or equal to 3.5: $M \geq M_0$, $M_0 = 3.5$. The values of the (τ, r) parameters were selected equal to: $\tau = 50, 100, 200$ and 400 days; $r = 0, 0.5, 1.0, 1.5$. The computations were made for both the initial subcatalogues, corresponding to the selected subregions (together with aftershocks) and only for the principal tremors for the considered subregions, after excluding aftershocks from the entire catalogue. The following procedure was used for excluding aftershocks [*Knopoff et al.*, 1982]. Assume that $\{t_m, M_m\}$ is the principal tremor. The $\{t_j, M_j\}$ events are the aftershocks of this tremor and are eliminated from the catalogue if:

1) $M_j < M_m$;
2) $t_m < t_j < t_m + T(M), T(M) = T_0 \times 10^{a(M-M_*)}$;
3) $d(m,j) < R(M), R(M) = R_0 \times 10^{b(M-M_*)}$.

Here $d(m,j)$ is the distance between the epicenters of events "m" and "j"; $T(M)$ and $R(M)$ are the dependencies on the magnitude M for the time and space windows for discriminating aftershocks; T_0, R_0, a, b, M_* are the parameters of these dependencies. Adhering to [*Knopoff et al.*, 1982], in the computations it was assumed that $T_0 = 30$ days; $R_0 = 10$ km; $a = b = 0.5$; $M_* = 4$. By definition, the first event in the catalogue is the principal tremor.

The number of events with a magnitude greater than or equal to 3.5 for the selected subregions before and after eliminating aftershocks respectively was: $Q_1 - 736, 370$; $Q_2 - 472, 217$; $Q_3 - 215, 60$; $Q_4 - 989, 223$; $Q_5 - 242, 160$; $Q_6 - 356, 108$; $Q_7 - 299, 165$; $Q_8 - 791, 365$.

The article gives the results of computations for values of the time constant $\tau = 100$ days (as being the most "characteristic" of the sampled τ values) and the parameter $r = 0, 0.5$ and 1.0. The results for other pairs of (τ, r) values are not cited here due to the limited size of the article and the absence of qualitative differences from the cited variants.

We note that the r parameter, defining the dependence of the contribution of an event to the total intensity of a point process in the considered region on magnitude (formulas (1), (2)), determines that weight which is assigned to one event or another. For $r = 0$ all the events are on an equal basis relative to their magnitudes and since there are more weak events than strong events the resulting influence matrices for the most part reflect the correlations between weak events. With an r increase the weight of strong events increases exponentially and therefore the influence matrices for "large" r reflect the correlations between strong events. In our analysis the results obtained for $r = 0$ will be called influence matrices for weak events, for $r = 0.5$ — for events of "average" intensity, and for $r = 1.0$ — for "strong" events.

Tables 1–3 give the values of elements of the influence matrices $(k_{\mu}^{(\beta)}, k_{\alpha}^{(\beta)})$ $\alpha = 1, \ldots, 8; \beta = 1, \ldots, 8$ for values of the parameter $r = 0, 0.5$ and 1.0 respectively; tables

Table 3a. Initial subcatalogues, $M_0 = 3.5, r = 1, \tau =$ 100 days

	0	1	2	3	4	5	6	7	8
1	.742	.239	.000	.000	.018	.000	.000	.000	.000
2	.417	.007	.571	.000	.005	.000	.000	.000	.000
3	.205	.000	.000	.788	.003	.000	.000	.000	.004
4	.129	.000	.000	.000	.871	.000	.000	.000	.000
5	.547	.000	.000	.000	.000	.443	.010	.000	.000
6	.122	.076	.000	.000	.000	.000	.803	.000	.000
7	.511	.016	.008	.000	.000	.000	.000	.465	.000
8	.546	.000	.000	.000	.003	.000	.000	.000	.451

Table 3b. Principal tremors, $M_0 = 3.5, r = 1, \tau = 100$ days

	0	1	2	3	4	5	6	7	8
1	.674	.307	.000	.000	.000	.000	.000	.000	.018
2	.829	.000	.171	.000	.000	.000	.000	.000	.000
3	.718	.000	.000	.100	.000	.024	.000	.000	.158
4	.658	.000	.000	.000	.342	.000	.000	.000	.000
5	.773	.000	.000	.000	.000	.221	.000	.006	.000
6	.700	.000	.000	.000	.000	.101	.140	.060	.000
7	.581	.000	.000	.000	.043	.000	.000	.350	.027
8	.771	.000	.000	.000	.000	.000	.000	.000	.229

with the designation "a" correspond to the initial subcatalogues (together with aftershocks), whereas those designated "b" correspond to the principal tremors for the selected subregions. The value of the τ parameter is always 100 days. The matrix columns are numbered from 0 to 8, and the rows from 1 to 8. The column with the heading "0" corresponds to the values of the stochasticity coefficients $k_\mu^{(\beta)}$. The heavy print designates values of the matrix elements greater than or equal to 0.1 (that is, 10% of the contribution to the mean intensity for the region).

Discussion of Results

First we will turn attention to Tables 1a, 2a, 3a, applying to series of events together with aftershocks. As might be expected, in this case each subregion was "closed in itself" for all values of the r parameter: all the matrix elements corresponding to the reciprocal influence of the subregions are negligible (an exception is the insignificant influence of Q_1 on Q_6 in Table 2a). For $r = 0$ the dominant contribution to mean intensity is from the self-exciting component, which is natural due to the strong grouping of weak events in the aftershock series. With an increase in r it is the strong events which begin to predominate, which results in an increase in the stochastic component at the expense of a decrease in the self-exciting component (but the effect of interaction of subregions is not manifested).

If Tables 1b, 2b, 3b, applying to interaction of series of principal tremors, are now examined, it is possible to trace there the same trend with an r increase: an increase in the contribution of the stochastic component at the expense of a decrease in other matrix elements. However, here we see a considerable interaction of subregions. The most diverse interaction is observed when $r = 0$ (Table 1b, correlation for weak events). It is interesting to note that an r increase results only in the elimination of some influences, but not the appearance of new ones.

For an analysis of Tables 1b, 2b and 3b we will prepare an auxiliary table in which we enter only the influence of subregions for the principal tremors pertinent to the elements $k_\alpha^{(\beta)}, \alpha, \beta = 1, \ldots, 8, \alpha \neq \beta, k_\alpha^{(\beta)} \geq 0.1$ ($\geq 10\%$).

$r = 0$	$r = 0.5$	$r = 1$
$Q_2 \longrightarrow Q_3$	$Q_2 \longrightarrow Q_3$	
$Q_2 \longrightarrow Q_6$	$Q_2 \longrightarrow Q_6$	
$Q_5 \longrightarrow Q_2$	$Q_5 \longrightarrow Q_3$	
$Q_5 \longrightarrow Q_3$		
$Q_5 \longrightarrow Q_4$	$Q_5 \longrightarrow Q_6$	$Q_5 \longrightarrow Q_6$
$Q_5 \longrightarrow Q_6$		
$Q_7 \longrightarrow Q_1$	$Q_7 \longrightarrow Q_6$	
$Q_7 \longrightarrow Q_6$		
$Q_7 \longrightarrow Q_8$		
$Q_8 \longrightarrow Q_3$	$Q_8 \longrightarrow Q_3$	$Q_8 \longrightarrow Q_3$
$Q_8 \longrightarrow Q_4$		
$Q_8 \longrightarrow Q_7$	$Q_8 \longrightarrow Q_7$	

The most "stable" relative to an r increase were the influence of adjacent subregions $Q_5 \longrightarrow Q_6$ and the remote influence $Q_8 \longrightarrow Q_3$ (Southern California, Parkfield region). The subregions can be arbitrarily divided into "aggressive" (inclined to an influence on adjacent and even remote regions) — these are Q_5 and Q_8, and "unaggressive" — Q_1, Q_3, Q_4, Q_6. The subregions Q_3 and Q_6 can be called the most "sensitive" because each of them "responds" to earthquakes from three other regions. It also is possible to define "autonomous" regions, that is, those which are least inclined both to exert an influence on other subregions and to be the object of such an influence — these are the subregions Q_1 and Q_4. The arrows in the figure show the interactions indicated in Table 1b.

It is interesting to compare Table 1a with 1b, 2a with 2b and 3a with 3b. The exclusion of aftershocks in a general case should result in a decrease in the self-exciting component (due to elimination of the strong grouping of aftershocks). This in actuality is observed in all regions (and the decrease is very strong, by several times), except for the Q_1 region, where the elimination of aftershocks results in an increase in the self-excitation coefficient. This effect is attributable to the fact that in late 1976 and late 1980 in the Q_1 subregion there were strong earthquakes which were accompanied by a large number of aftershocks. As a result of elimination of aftershocks two long time intervals of absence of principal tremors were formed in the Q_1 subregion; a special feature of the seismic regime of the Q_1 subregion is that in other intervals a quite high intensity of the principal tremors persisted. As a result, a strong grouping of principal tremors was obtained which was reflected in an increase in the self-excitation coefficient.

The fact that earthquakes in one of the subregions increase the probability of earthquakes in another and vice versa (for example, such as $Q_7 \longrightarrow Q_8$ and $Q_8 \longrightarrow Q_7$) can be interpreted as the dependence of earthquakes in these subregions on the overall tectonic structure. It is evidently feasible to combine subregions Q_7 and Q_8.

It is more difficult to interpret the unidirectional influence of earthquakes of one subregion on the earthquakes of another subregion without a reciprocal influence. Most of the interactions are of this type. They could arise due to the directional influence of disturbing tectonic forces from the direction of one subregion to another adjacent subregion (for example, the influence of subregion Q_5 on the adjacent subregions $Q_3, Q_4, Q_6; Q_2 \longrightarrow Q_3; Q_7 \longrightarrow Q_6$). It is more difficult to explain how the earthquakes in one subregion can exert an influence on events in another subregion not adjacent to it ($Q_2 \longrightarrow Q_6, Q_5 \longrightarrow Q_2, Q_7 \longrightarrow Q_1$, $Q_8 \longrightarrow Q_3$). Evidently further studies of the seismic regime and geological structure of the region are necessary in order to construct a physicomechanical model of the tectonic interaction of lithospheric structural elements.

Finally, a study was made of the problem of estimating the dispersion of the determined values of the parameters (7). As already mentioned, they can be determined approximately by computing the matrix of second derivatives of the function (3) for the parameters (7) (which is easily computed using (9)), its inversion and examination of the square roots of the diagonal elements of this inverse matrix. Strictly speaking, such an analysis is justified for the β-th region only with high N_β values. In our case the N_β values vary from 60 to 989. Accordingly, the dispersions behave approximately as $1/\sqrt{N_\beta}$; the greatest dispersions are observed for an evaluation of the stochasticity coefficients. The standard deviations for the stochasticity coefficients vary from 0.14 for $N_\beta = 989$ to 0.91 for $N_\beta = 60$. We will cite the values of the standard deviations of the coefficients for subregion Q_1, having the intermediate number $N_\beta = 370$ for the principal tremors. The values of the coefficients are given in the first line of Table 2b. Together with the standard deviations they look as follows: 0.27 ± 0.22; 0.71 ± 0.16; 0 ± 0.09; 0 ± 0.18; 0 ± 0.12; 0 ± 0.15; 0 ± 0.10; 0 ± 0.12; 0.02 ± 0.11.

We see that in a typical situation the standard deviations are close to 0.1, so that the condition which we adopted (assume that the influence coefficients are significant if they exceed 0.1) can be considered to a certain degree to be sound. However, as indicated by the examples cited above, with small N_β values only coefficients appreciably exceeding 0.1 can evidently be considered reliable.

The proposed approach to an analysis of the seismic regime by constructing influence matrices makes possible a quantitative formalization of the search for the statistical correlations between different seismically active regions. The developed approach may be useful in the search for remote precursors of strong earthquakes (when choosing "signal" regions).

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