Synchronization of multi-fractal parameters of regional and global low-frequency microseisms

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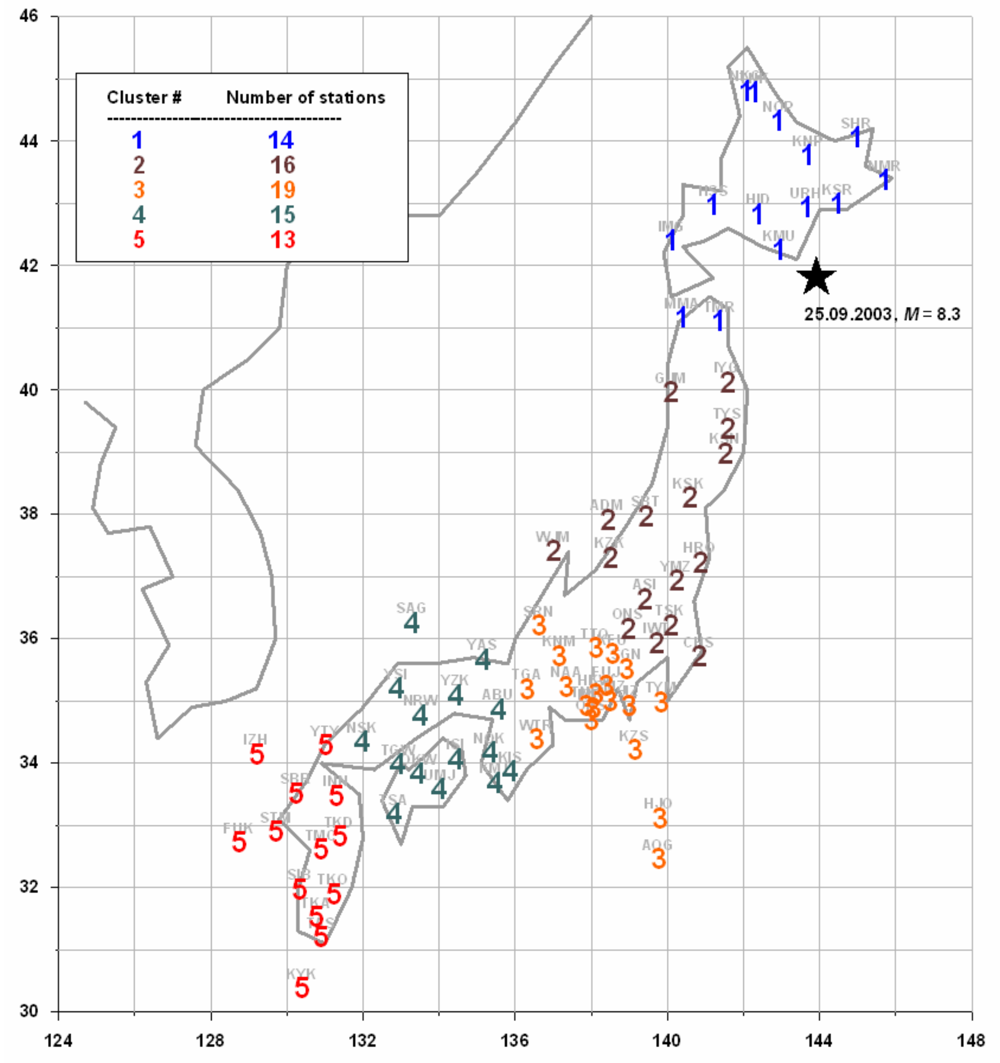
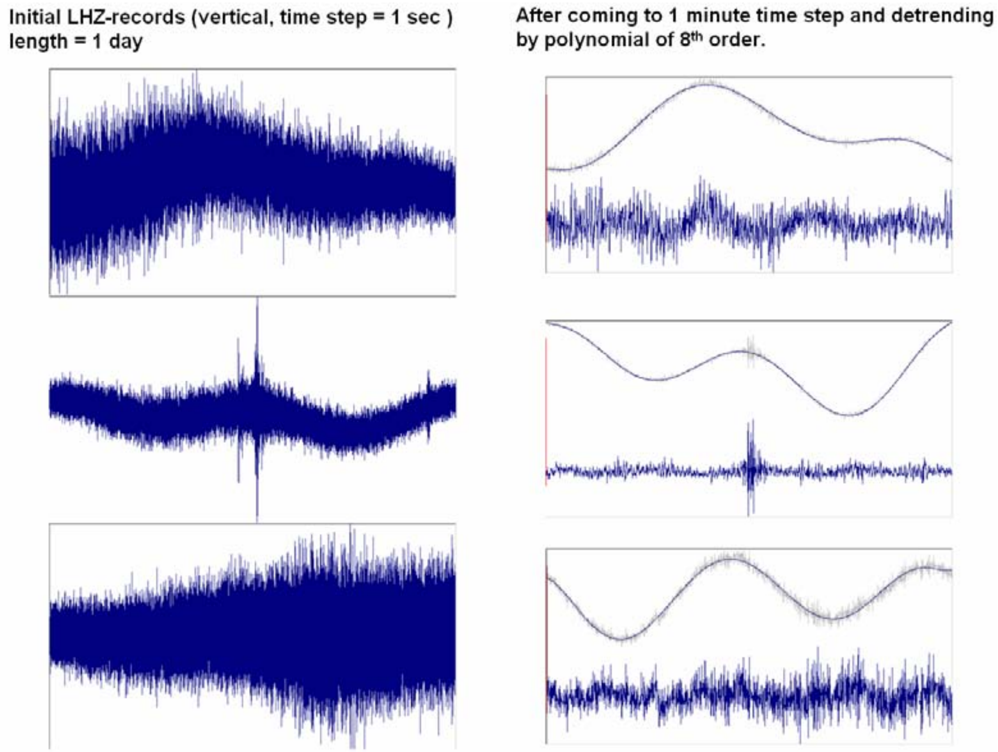
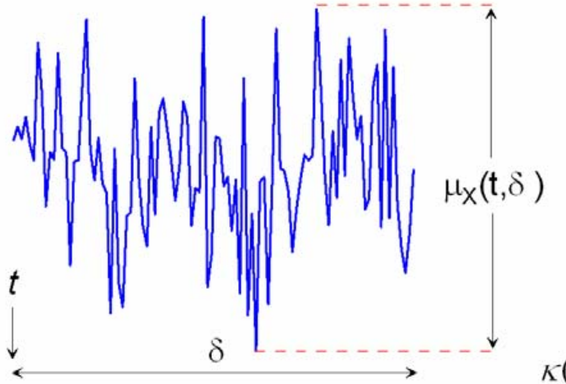


Positions of 77 F-net broad-band seismic stations and their splitting into 5 clusters. Star indicates hypocenter of Hokkaido earthquake, 25.09.2003, M=8.3.



Examples of initial seismic records with 1 Hz sampling rate of 1 day length (left panel) and results of their transform to 1 minute time sampling step and removing trend by polynomial of the 8th order (right panel).

Scale-dependent measure of the signal variability:



$$\mu_X(t,\delta) = \max_{t\le s\le t+\delta} X(s) - \min_{t\le s\le t+\delta} X(s)$$

$$M_X(\delta,q) = <\mu_X^q(t,\delta)>_t$$

$$\kappa(q) = \lim_{\delta\to 0} \frac{\ln M_X(\delta,q)}{\ln|\delta|}$$

$$M_X(\delta,q) \sim |\delta|^{\kappa(q)}, \delta \to 0$$

Mono-fractal signal:

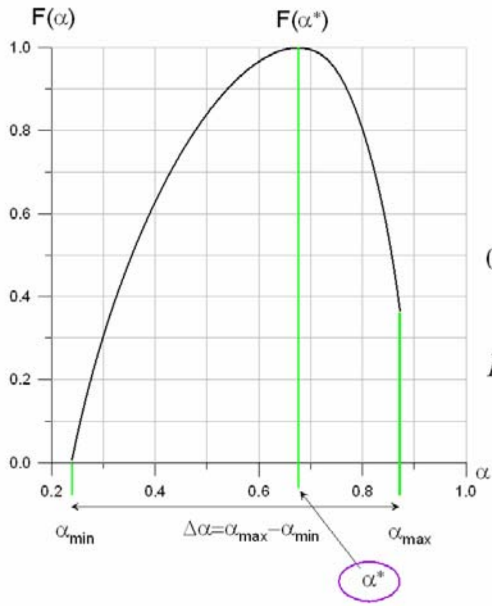
$$\kappa(q) = Hq, H = const, 0 < H < 1$$

H – Hurst exponent

Multi-fractal signal: $\kappa(q) = qh(q)$

Singularity spectrum:

$$F(\alpha) = \max\{\min_q(\alpha q - \tau(q)), 0\}; \quad \tau(q) = \kappa(q) - 1 = qh(q) - 1$$



Singularity spectra parameters:

$$\alpha_{\min}, \alpha_{\max}, \quad \Delta\alpha = \alpha_{\max} - \alpha_{\min},$$

$$\alpha^*, \quad F(\alpha^*) = \max_\alpha F(\alpha)$$

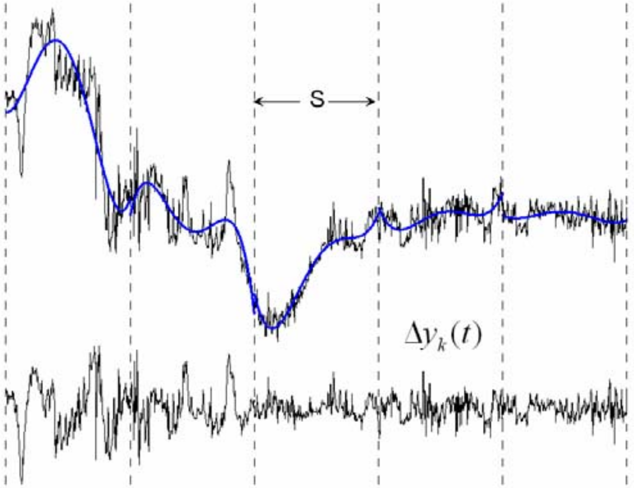
α^* - generalized Hurst exponent

$0 < F(\alpha^*) \le 1$ - fractal dimensionality of the measure support

$F(\alpha)$ - fractal dimensionality of time moments *t* set for which:

$$\mu_X(t,\delta) \sim |\delta|^\alpha, \delta \to 0$$

$$\mu_X(t,\delta) = \max_{t\le s\le t+\delta} X(s) - \min_{t\le s\le t+\delta} X(s)$$

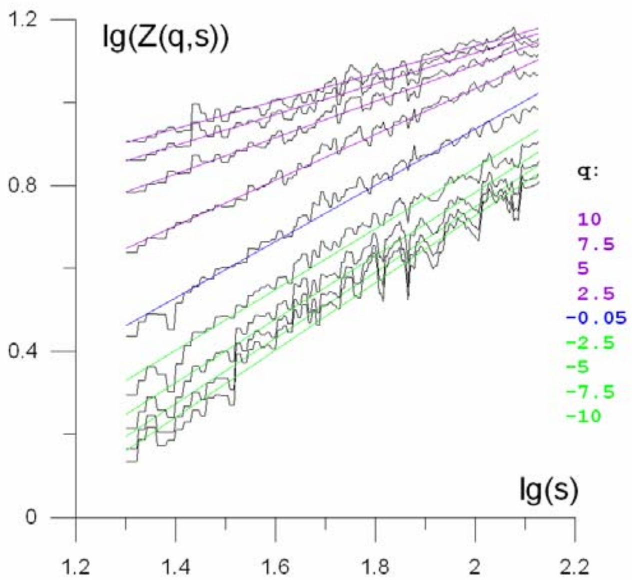


Detrended fluctuation analysis (DFA) for estimating singularity spectra

Local polynomial trends within non-overlapping "small" intervals of the length S samples.

Deflections from local polynomial trends within "small" sub-intervals of the length S.

$$Z(q,s)=\left(\sum_{k=1}^{[N/s]}(\max_{1\le t\le s}\Delta y_k(t)-\min_{1\le t\le s}\Delta y_k(t))^q \Big/ [N/s]\right)^{1/q}$$



$$Z(q,s)\sim s^{h(q)}$$

$h(q)$ - a slope coefficients of linear trends: $\lg(Z(q,s))\approx h(q)\cdot\lg(s)+const$

Linear Predictability Index

Trivial predictor 1 – step ahead by previous n samples: $\hat{x}_0(t+1)=\sum_{s=t-n+1}^{t}x(t)/n$

$V_0=\mathrm{var}(\varepsilon_0)=\sum_{t=n+1}^{N}\varepsilon_0^2(t)/(N-n),\quad n<N\quad where\quad \varepsilon_0(t)=x(t)-\hat{x}_0(t)$

$N=1440$ (1 *day*); $n=60$ (1 *hour*);

AR(2) *predictor* 1 – *step ahead*: $\hat{x}_{AR}(t+1)=a_1x(t)+a_2x(t-1)+d$

Vector of AR(2) – *parameters* $c=(a_1,a_2,d)^T$ *is defined by least squares approach by previous n samples*:

$\hat{c}(t)=A^{-1}(t)\cdot R(t),\quad A(t)=\sum_{s=t-n+3}^{t}Y(s)\cdot Y^T(s),\quad R(t)=\sum_{s=t-n+3}^{t}x(s)\cdot Y(s),$

where: $Y(t)=(x(t),x(t-1),1)^T$

$V_{AR}=\mathrm{var}(\varepsilon_{AR})=\sum_{t=n+3}^{N}\varepsilon_{AR}^2(t)/(N-n-2)\quad where\quad \varepsilon_{AR}(t)=x(t)-\hat{x}_{AR}(t)$

$\rho=V_0/V_{AR}-1$; *for linearly predicted series* $\rho>0$.

Multiple robust correlation measure

Let $u_r(t),\ r=1,\dots,m,\ s=1,\dots,N$ – *multiple time series, t – time index.*

Let's present p – th component as a sum:

$u_p(t)=w_p(t)+\varepsilon_p(t),\quad w_p(t)=\sum_{r=1,\ r\neq p}^{N}\gamma_r^{(p)}\cdot u_r(t),\quad 1\le p\le m$

$\gamma_r^{(p)}:\quad \sum_{s=1}^{N}|\varepsilon_p(t)|=\sum_{s=1}^{N}|u_p(t)-\sum_{r=1,\ r\neq p}^{N}\gamma_r^{(p)}\cdot u_r(t)|\to\min_{\gamma_r^{(p)}}$

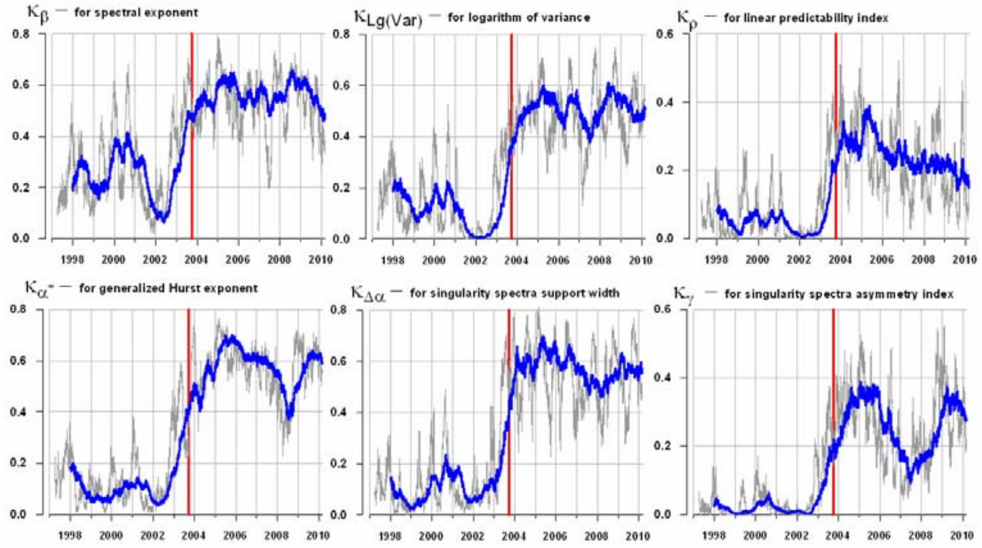
Robust canonical correlation of p – th component μ_p:

$$\mu_p=\frac{S(\varphi_p^2)-S(\psi_p^2)}{S(\varphi_p^2)+S(\psi_p^2)},\quad \varphi_p(t)=\frac{u_p}{S(u_p)}+\frac{w_p}{S(w_p)},\quad \psi_p(t)=\frac{u_p}{S(u_p)}-\frac{w_p}{S(w_p)}$$

where: $S(\xi)=med|\xi-med(\xi)|$ *is an absolute median deviation of* ξ.

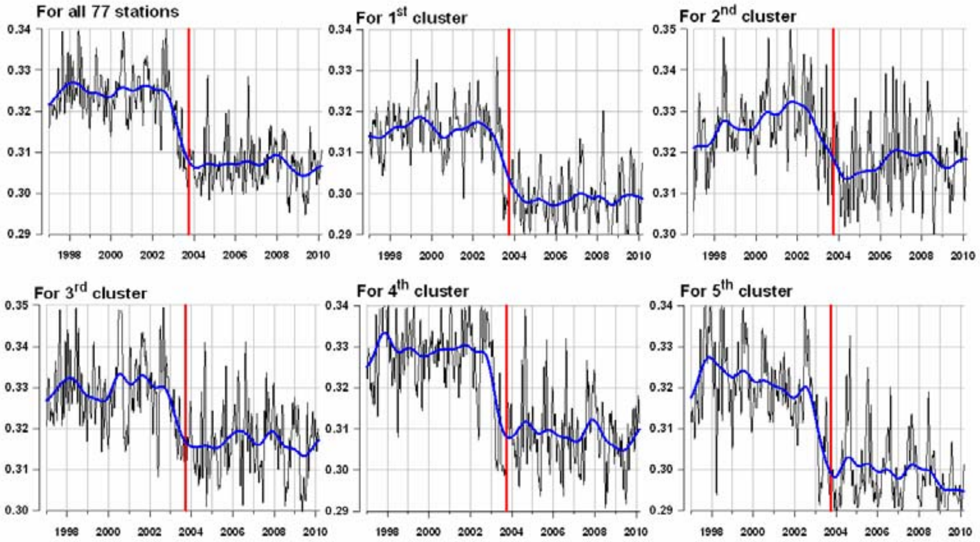
Multiple correlation measure: $\kappa=\prod_{p=1}^{Q}|\mu_p|,\quad 0\le\kappa\le 1$

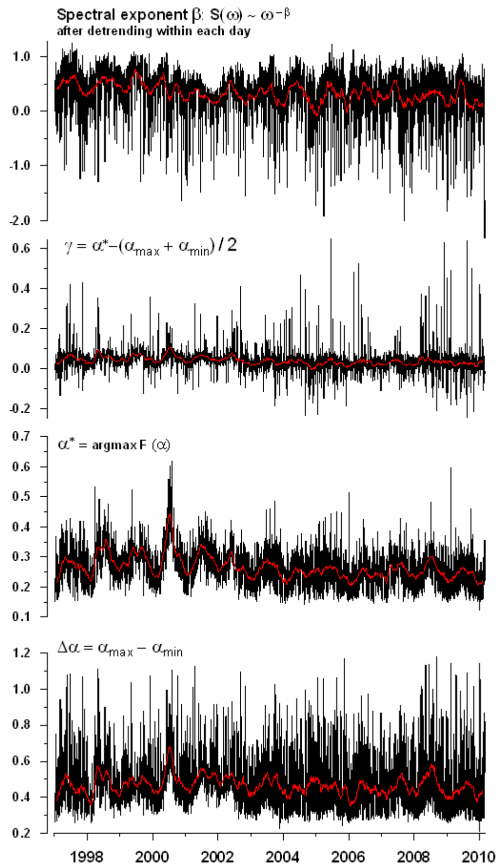
Multiple correlation measures K (product of abs. canonical correlations) estimated within moving time window of the length 91 days (0.25 year - grey lines) and 365 days (1 year - bold blue lines) for increments of variations of medians of daily estimates of different statistical parameters of seismic records within adjacent 1 day time intervals for 1-minute data for 5 spatial clusters of stations.
Vertical red lines indicate Hokkaido earthquake, M = 8.3, Sept 25, 2003.



Right-hand end of moving time windows of the length 0.25 (grey lines) and 1 year (bold blue lines)

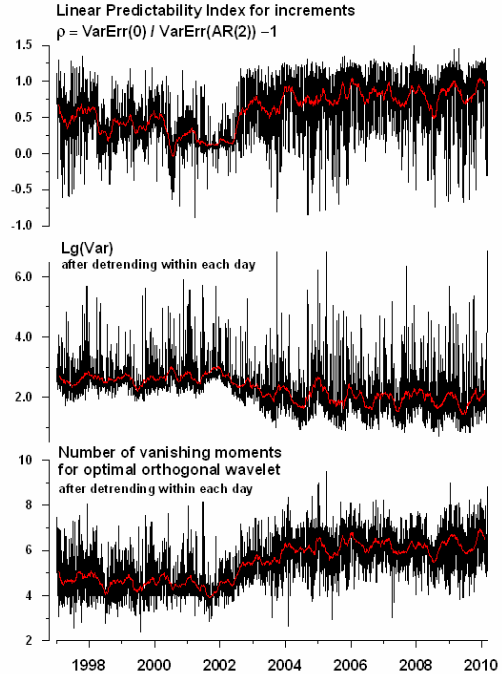
Averaged curves for variations of median values of multi-fractal singularity spectra parameter $\Delta\alpha$, estimated within time windows of 30 minute length for initial LHZ-records with 1 sec sampling for all F-net stations and separately for 5 clusters of stations.
Thin black lines - Gaussian kernel smoothing with radius 13 days.
Bold blues lines - Gaussian kernel smoothing with radius 0.5 year.
Vertical red lines indicate Hokkaido earthquake, M = 8.3, Sept 25, 2003.

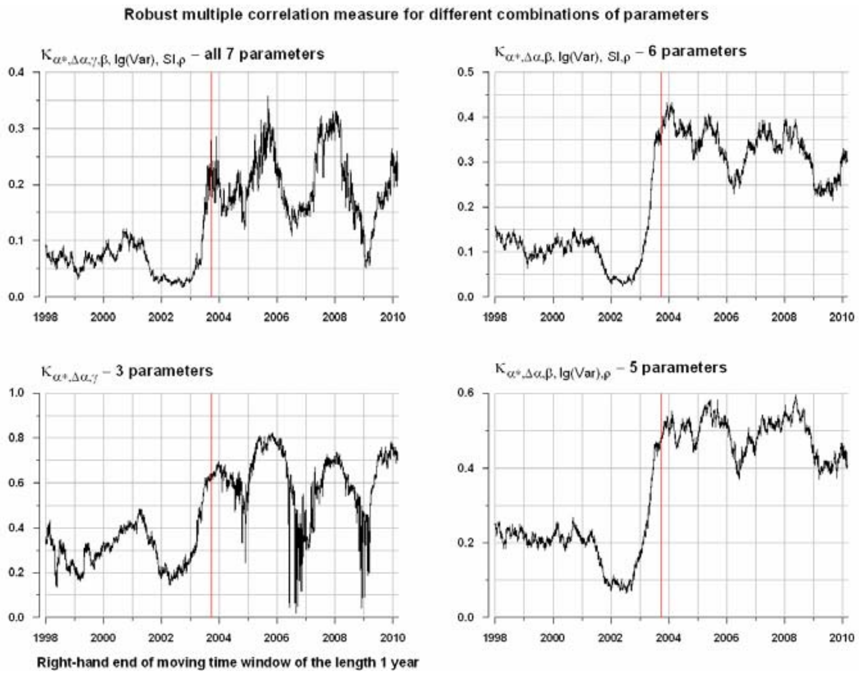


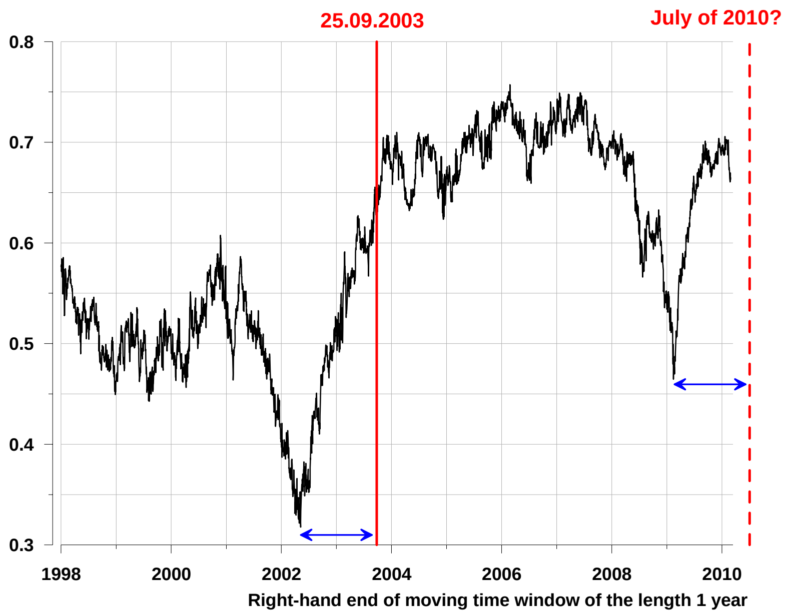


Variations of medians (for all stations) of 7 statistics estimated within adjacent time windows of the length 1 day for seismic records after coming to 1 minute sampling by averaging initial 1 Hz data.

Red lines - 57 days running average.







Squared robust correlation between median values of $\Delta\alpha$ and α^* taken over all F-net network stations.

Cluster analysis of 7D clouds of vectors of parameters within 2 year moving window

$\vec{\xi} = (\Delta\alpha, \alpha^*, \gamma, \lg(Var), \beta, SI, \rho)$ – *their medians each day*

$\vec{\xi}^{(t)}$ – *7D vectors within current time window*, $t = 1, \dots, N = 730$

$<\xi_k> = \sum_{t=1}^{N} \xi_k^{(t)} / N, \quad s_k^2 = \sum_{t=1}^{N} (\xi_k^{(t)} - <\xi_k>)^2 / (N-1)$ – *estimates of mean and st.dev.*

Normalization: $\zeta_k^{(t)} = (\xi_k^{(t)} - <\xi_k>)/s_k, \quad k = 1, \dots, m, \quad m = 7;$

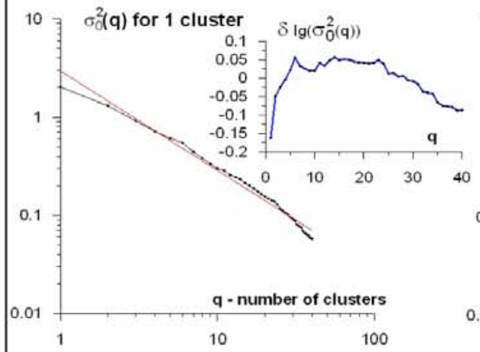
$\Gamma_r, r = 1, \dots, q$ – *split N vectors* $\vec{\zeta}^{(t)}$ *within moving window to q clusters*; $2 \le q \le 40$

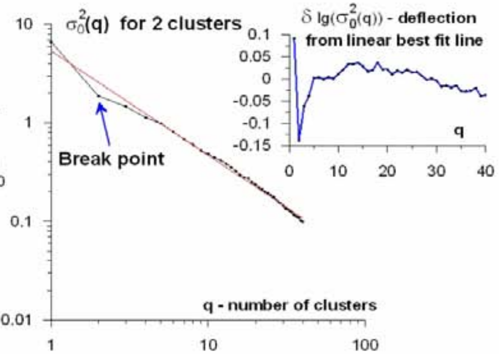
$\vec{z}^{(0)} = \sum_{t=1}^{N} \vec{\zeta}^{(t)} / N$ – *vector of general mean value;*

$\vec{z}^{(r)} = \sum_{\zeta \in \Gamma_r} \vec{\zeta}^{(t)} / n_r$ – *vector of mean value of cluster* Γ_r; $\sum_{r=1}^{q} n_r = N$

$$PFS(q) = \frac{\sigma_1^2}{\sigma_0^2}, \quad \sigma_0^2(q) = \frac{\sum_{r=1}^{q} \sum_{\zeta \in \Gamma_r} |\vec{\zeta} - \vec{z}^{(r)}|^2}{N - q}, \quad \sigma_1^2(q) = \frac{\sum_{r=1}^{q} \nu_r \cdot |\vec{z}^{(r)} - \vec{z}^{(0)}|^2}{q-1}, \quad \nu_r = \frac{n_r}{N}$$

Cases of 1 and 2 clusters are distinguished by existence of break point at q=2



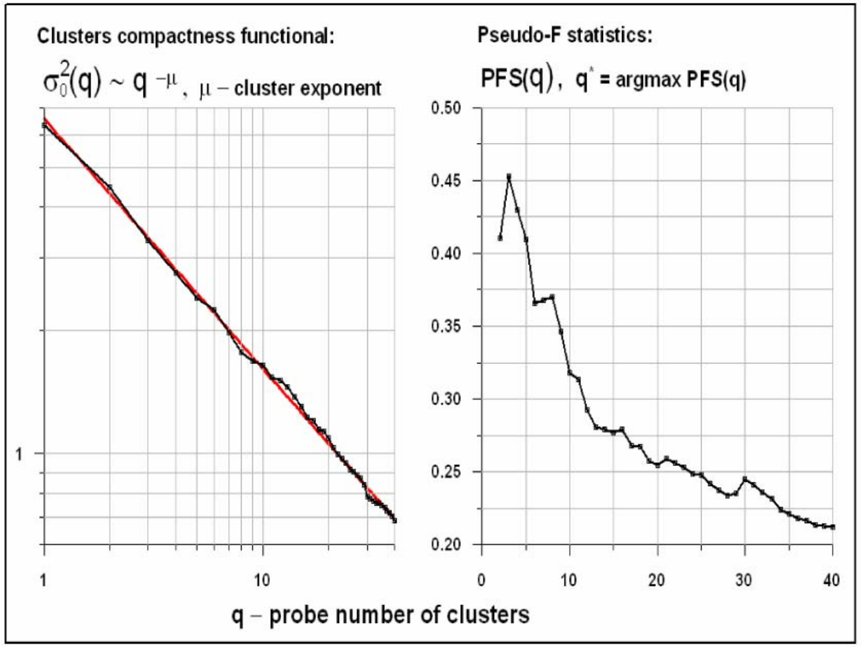


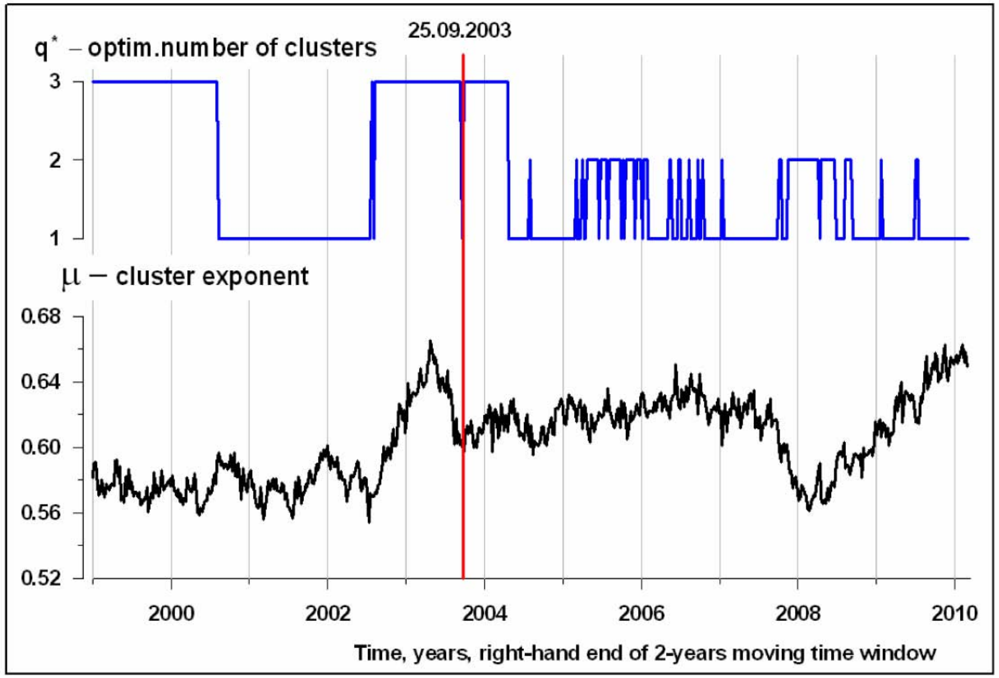
Let $q_0 = \arg\max_{2 \le q \le m_C} PFS(q)$

If $q_0 > 2$ *then* $q^* = q_0$

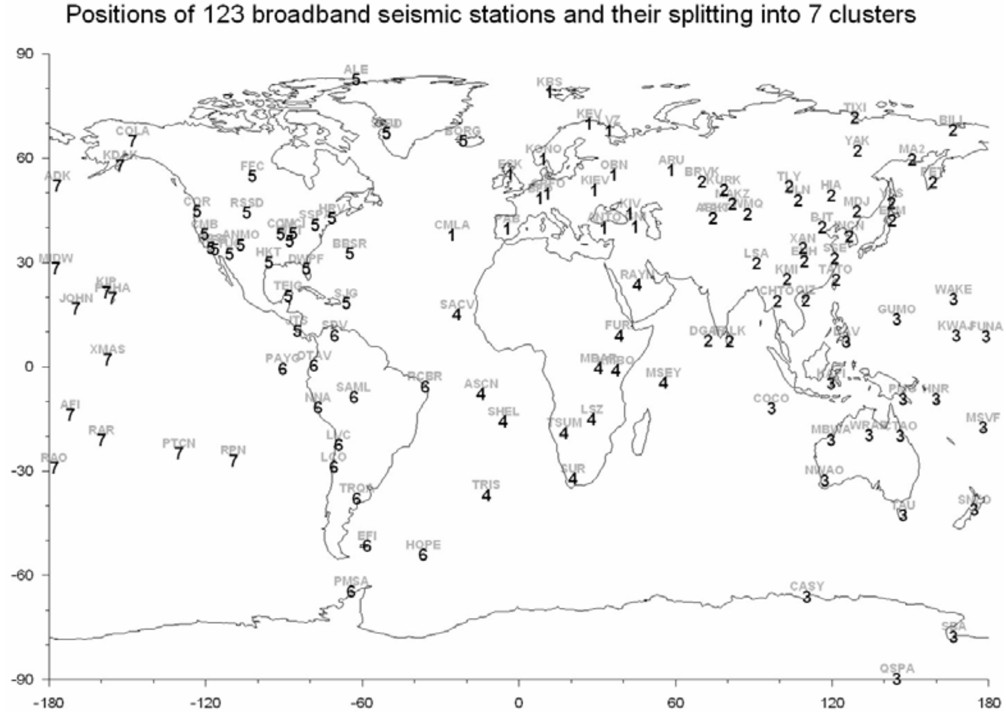
else if $\delta \log(\sigma_0^2(1)) \le \max_{2 \le q \le m_C} \delta \log(\sigma_0^2(q))$ *then* $q^* = 1$

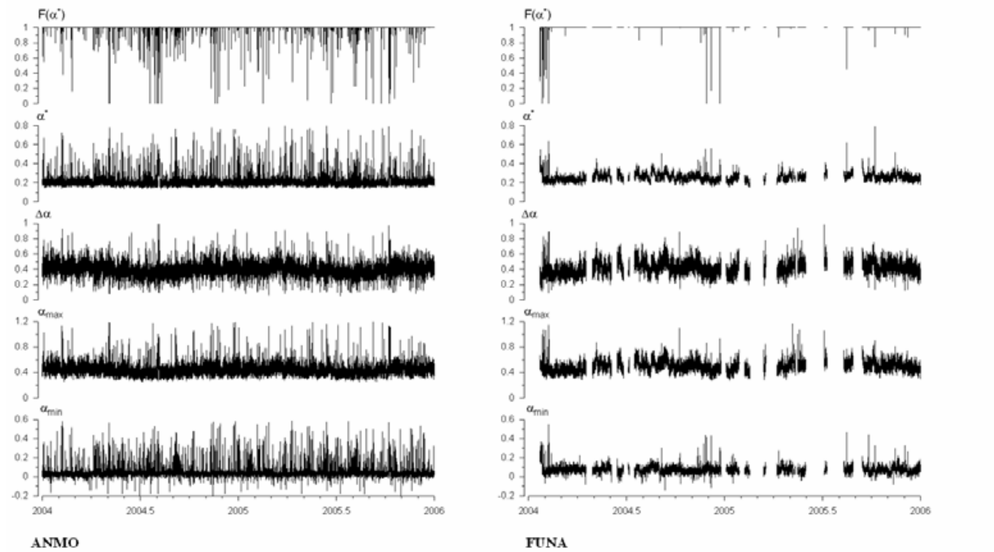
else $q^* = 2$



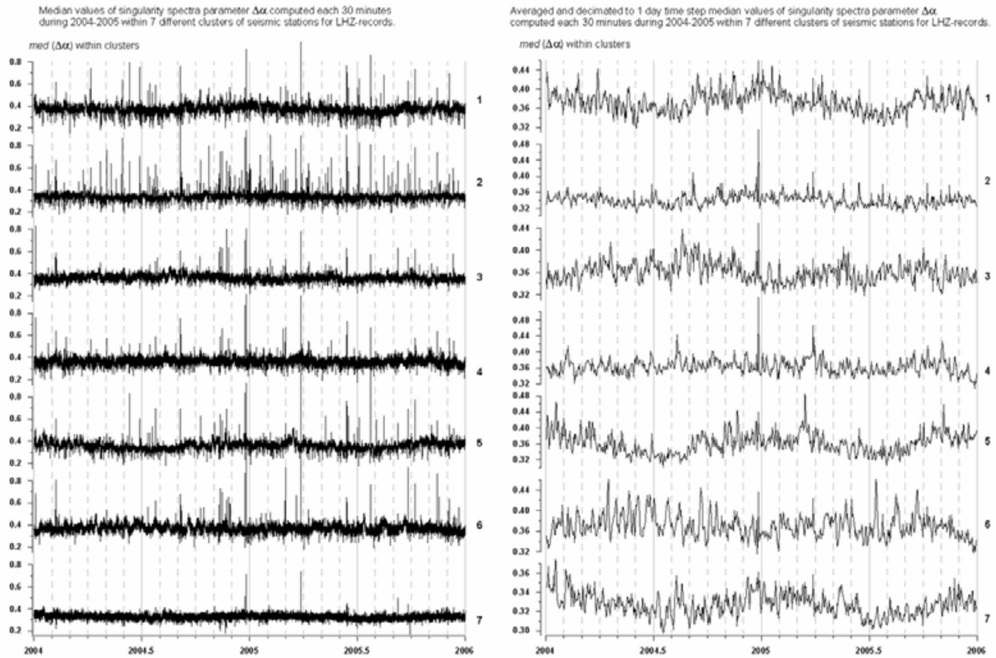


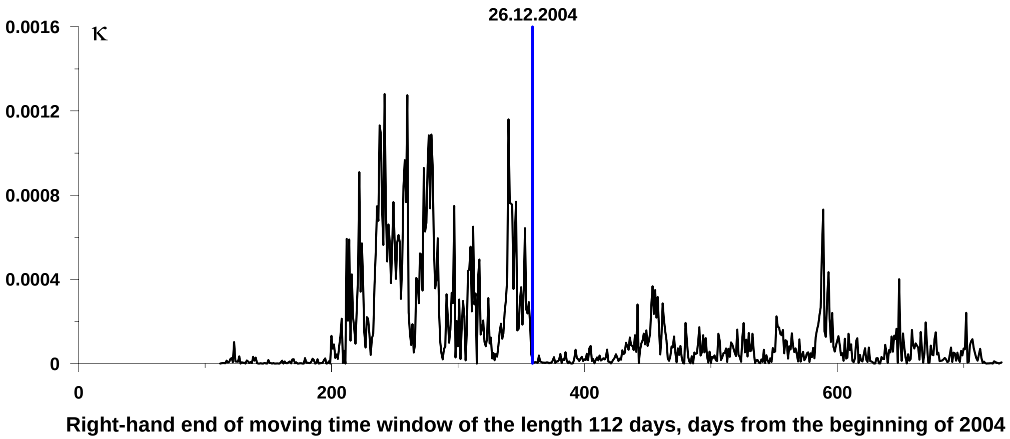
Optimal number of clusters and cluster exponent estimates





Examples of variations of multi-fractal singularity spectra parameters, estimated within adjacent time windows of the length 1800 samples (30 minutes) for LHZ records for station ANMO (Albuquerque, New Mexico, USA) and FUNA (Funafuti, Tuvalu, Pacific ocean).





Multiple robust correlation measure for averaged and decimated to 1 day time step median values of singularity spectra parameters $\Delta\alpha$ computed each 30 minutes during 2004-2005 within 7 different clusters of seismic stations for LHZ-records.

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Conclusion:

The low-frequency microseisms field at Japan Islands transfers to high level synchronization of its parameters starting from the middle of 2002, one year before the Hokkaido earthquake, 25 of September, 2003, M=8.3. This high level of synchronization keeps rather constant up to the current time. Based on the well-known statement of the theory of catastrophes that synchronization is one of the flags of an approaching catastrophe, it may be suggested that the Hokkaido event, notwithstanding its power (M = 8.3), could be only a foreshock of a still stronger earthquake forming in the region of Japan's islands. The cluster analysis of 7 median daily statistics from the whole network indicates a strong linear trend of cluster exponent μ starting from 2007 which is continuing till now. This trend peculiarity is similar to the trend before 2003 event. The peculiarities of squared correlation coefficient estimate within 1 year moving time window between daily median values of multi-fractal singularity spectra parameters $\Delta\alpha$ and α^* indicates that starting from July of 2010 Japan Islands come to state of waiting strong earthquake.

A synchronization of global microseisms multi-fractal parameters was observed during 160 days prior to catastrophic Sumatra earthquake, M=9.1, 26 of December, 2004.

References.

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