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SCIENTIFIC COMMUNICATIONS

Analysis of Canonical Coherences in the Problems of Geophysical Monitoring

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Lyubushin [1, 2] proposed an approach to the synthetic analysis of long routine observation series of background variations in geophysical characteristics (crustal deformations, groundwater levels in wells, groundwater concentrations of chemical elements, gas emanation intensity, etc.) synchronously measured by a spatial monitoring network. The purpose of such observations is the detection of the so-called *synchronization signal* of these variations (possibly differing in their physical origin) and determination of the duration and frequency range of such a signal. From the algorithmic standpoint, the method consists in the estimation of the moving time-window variation in the eigenvalue of the spectral matrix and, in fact, is a multidimensional extension of the 1D spectral-time analysis (STA), widely used in geophysics. The synchronization signal is associated with bursts of (increases in) the maximum eigenvalue. Such a signal can serve as a precursor of strong earthquakes [1–3].

Within the framework of the method offered in [1, 2], this work develops the apparatus of *canonical coherences* applied to the statistical analysis of multivariate time series of monitoring systems. The goal is to construct a frequency-dependent coherence measure between two temporal vector series representing the measurements of two different geophysical fields at two different points and to analyze the time variation of this characteristic. The canonical coherences provide a means for detecting the synchronization signal, as an alternative to the method offered in [1, 2] for the evolution analysis of the spectral matrix eigenvalue.

INTRODUCTION

The squared modulus of the coherence spectrum, widely used as a coherence measure between two scalar time series $x(t)$ and $y(t)$, is defined as the following function of frequency ω, taking on values from 0 to 1:

$$\gamma_{xy}^2(\omega) = |S_{xy}(\omega)|^2/(S_{xx}(\omega)S_{yy}(\omega)), \qquad (1)$$

where S_{xx} and S_{yy} are the power spectra of the series $x(t)$ and $y(t)$, and S_{xy} is their cross spectrum.

However, the need often arises to construct a function describing the frequency-dependent coherence measure between vector series $\mathbf{X}(t)$ and $\mathbf{Y}(t)$. For example, these series may represent the synchronous measurements of two different physical parameters at two different points of space. The number of the vector components of $\mathbf{X}$ and $\mathbf{Y}$ may be different depending on the number of the monitoring stations recording the x and y variations.

A characteristic similar to (1) is also useful, if measurements are made at two stations recording variations in many parameters, which combine to form the vector series $\mathbf{X}(t)$ and $\mathbf{Y}(t)$.

The ordinary correlation coefficient between two scalar random values was extended, under the name of canonical correlations, to the vectors of various dimensions by Hotelling in his classical work on the multidimensional statistical analysis [4]. This approach is discussed in detail in [5, 6].

The extension of the canonical correlations to the case of vector time series consists in the time-to-frequency domain transformation and replacement of the correlation matrices by the Hermitean complex spectral matrices. Correspondingly, the terminology changes: the canonical correlations are replaced by the frequency-dependent canonical coherences. This transition is described in [7, 8]. Below, we briefly clarify the meaning of the latter and present the main formulas for their calculation.

CANONICAL COHERENCES OF TWO VECTOR TIME SERIES

Let $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ be m- and n-dimensional time series, and let t be the discrete time corresponding to the consecutive measurements of the $\mathbf{X}$ and $\mathbf{Y}$ components. For the sake of definiteness and without loss of generality, we set $m \le n$. Thus, the multivariate series $\mathbf{Z}(t)$ of the dimension $l = m + n$ is observed. From physical or some other considerations, it is convenient to subdivide this series into two nonoverlapping vector time series $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ and to analyze their mutual coherence at various frequencies and in various time intervals.

Let $\mathbf{S}_{xx}(\omega)$ and $\mathbf{S}_{yy}(\omega)$ be the quadratic, nonnegatively definite spectral $m \times m$ and $n \times n$ matrices of the respective series $\mathbf{X}(t)$ and $\mathbf{Y}(t)$, and let xy be the cross-spectral $m \times n$ matrix of $\mathbf{X}$ and $\mathbf{Y}$, $\mathbf{S}_{yx}(\omega) = = \mathbf{S}_{xy}^{H}(\omega)$, where H is the sign of the Hermitean conjugation [7, 8]. We consider the complex $m \times m$ matrix

$$\mathbf{U}(\omega) = \mathbf{S}_{xx}^{-1}(\omega)\mathbf{S}_{xy}(\omega)\mathbf{S}_{yy}^{-1}(\omega)\mathbf{S}_{yx}(\omega) \quad (2)$$

and $n \times n$ matrix

$$\mathbf{V}(\omega) = \mathbf{S}_{yy}^{-1}(\omega)\mathbf{S}_{yx}(\omega)\mathbf{S}_{xx}^{-1}(\omega)\mathbf{S}_{xy}(\omega). \quad (3)$$

It is easy to show that, for a 1D time series ($m = n = 1$), formulas (2) and (3) determine the ordinary squared modulus of coherence (1).

The proofs of all statements presented below concerning the eigenvalues and eigenvectors of matrices (2) and (3) can be found in Section 10 of [8].

First, the eigenvalues of matrices (2) and (3) are real and nonnegative. We arrange the eigenvalues of the matrix $\mathbf{U}(\omega)$ in the nonincreasing order,

$$1 \geq \mu_1^2(\omega) \geq \mu_2^2(\omega) \geq \ldots \geq \mu_m^2(\omega) \geq 0, \quad (4)$$

where $\mu_1^2(\omega)$ is the maximum eigenvalue of matrix (2) not exceeding 1. Then, the first m eigenvalues of matrix (3) coincide with those of (4), and the remaining $n - m$ eigenvalues are zero ($m \leq n$).

The application of m-dimensional filtering to the vector time series $\mathbf{X}(t)$ gives a scalar time series $\xi(t)$. This filtering may be represented as a sequence of the following operations. First, the discrete Fourier transformation is applied to $\mathbf{X}(t)$; its result is denoted as $\tilde{\mathbf{X}}(\omega)$, where ω is a discrete frequency. Then, the m-dimensional complex vector $\tilde{\mathbf{X}}(\omega)$ is scalarly multiplied by an m-dimensional complex vector $\mathbf{A}(\omega)$ representing the transfer function of an m-dimensional filter, $\tilde{\xi}(\omega) = \mathbf{A}(\omega)\tilde{\mathbf{X}}^{H}(\omega)$. Finally, the inverse discrete Fourier transformation is applied to the 1D sequence of complex numbers $\tilde{\xi}(\omega)$; the real part of the result will be denoted $\xi(t)$. A similar procedure involving an unknown n-dimensional filter $\mathbf{B}(\omega)$ is applied to the n-dimensional time series $\mathbf{Y}(t)$, which yields a scalar time series $\eta(t)$.

Now, the multidimensional filters $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ are chosen in such a way that the squared modulus of coherence (1) between the series $\xi(t)$ and $\eta(t)$ is maximal for each frequency ω. To satisfy this condition, the m-dimensional filter $\mathbf{A}(\omega)$ should be represented by the eigenvector $\mathbf{u}^{(1)}(\omega)$ of the complex $m \times m$-matrix (2), corresponding to the maximum eigenvalue $\mu_1^2(\omega)$, and the n-dimensional filter $\mathbf{B}(\omega)$ should be represented by the eigenvector $\mathbf{v}^{(1)}(\omega)$ of the complex $n \times n$-matrix (3), corresponding to the same eigenvalue. Moreover, the maximum value itself of the squared modulus of coherence between ξ and η is equal to $\mu_1^2(\omega)$. The resulting scalar time series will be denoted as $\xi^{(1)}(t)$ and $\eta^{(1)}(t)$; they are called the first canonical components of the original vector series $\mathbf{X}(t)$ and $\mathbf{Y}(t)$.

Besides the first canonical components, the second, third, etc., through mth ones are readily defined with the help of the eigenvectors $\mathbf{u}^{(2)}(\omega)$, $\mathbf{v}^{(2)}(\omega)$; $\mathbf{u}^{(3)}(\omega)$, $\mathbf{v}^{(3)}(\omega)$; …; $\mathbf{u}^{(m)}(\omega)$, $\mathbf{v}^{(m)}(\omega)$ of matrices (2) and (3); their pairwise squared moduli of coherence are $\mu_2^2(\omega)$, $\mu_3^2(\omega)$, …, $\mu_m^2(\omega)$.

Since the eigenvectors of matrix (2) are orthogonal, the canonical components $\xi^{(i)}$ and $\xi^{(j)}$ are uncorrelated (statistically independent for Gaussian time series) at noncoincident i and j. Similarly, the various components $\eta^{(i)}$ and $\eta^{(j)}$, as well as the pairs $\xi^{(i)}$ and $\eta^{(j)}$, are uncorrelated at noncoincident i and j.

Thus, the multidimensional case involves m coherences (4) rather than the single quadratic coherence (1), the maximum coherence $\mu_1^2(\omega)$ being most interesting and appropriate as a multidimensional analogue of (1).

CANONICAL COMPONENT COHERENCE

We consider the case of multidimensional observations when there is a l-dimensional time series $\mathbf{Z}(t)$. The series will not be divided into two different vector time sequences. The set of all components of the series will be analyzed.

The ith canonical component coherence $\nu_i^2(\omega)$ is defined here as the first canonical coherence $\mu_1^2(\omega)$ under the assumption that the time series $X(t)$ consists only of the ith component Z_i of the vector time series $\mathbf{Z}(t)$, and the time series $\mathbf{Y}(t)$ consists of all other components. Thus, $m = 1$, $n = l - 1$, and no canonical coherences, other than the first one, exist. In essence, the canonical component coherence $\nu_i^2(\omega)$ provides the frequency-dependent measure of the coherence between the variations of the scalar series $Z_i(t)$ and variations of all other components of the vector time series $\mathbf{Z}(t)$ under consideration.

The quantity $\mathbf{S}_{xx}(\omega)$ in (2) and (3) being a scalar now rather than matrix, the calculation of $\nu_i^2(\omega)$ is simplified as compared to (4). If $S_{zz}^{ij}(\omega)$, $i, j = 1, \ldots, l$ are the elements of the spectral matrix $\mathbf{S}_{zz}(\omega)$ of the l-dimensional time series $\mathbf{Z}(t)$, we have

$$\nu_i^2(\omega) = \sum_{j,k=1}^{l-1} h_j^{(i)}(\omega)A_{jk}^{(i)}(\omega)h_j^{(i)*}(\omega)/S_{zz}^{ii}(\omega), \quad i = 1, \ldots, l, \quad (5)$$

where the $(l - 1)$-dimensional complex vector $h_j^{(i)}(\omega)$, $j = 1, \ldots, (l - 1)$ is obtained by the removal of the ith element $\mathbf{S}_{zz}(\omega)$ from the ith row of the spectral matrix $S^{ii}_{zz}(\omega)$. The matrix $(A^{(i)}_{jk}(\omega))$, $i = 1, \ldots, l$; $j, k = 1, \ldots, (l-1)$) is the inverse of the spectral matrix $\mathbf{S}_{zz}(\omega)$ in which the ith row and ith column are removed. The sign "*" means complex conjugation.

Then, the set of the frequency-dependent measures $\nu_i(\omega)$, characterizing the coherence between the ith component and all other, is used for the construction of a statistic characterizing the coherence between the variations for the whole series:

$$\kappa(\omega) = \nu_1(\omega)\nu_2(\omega)\ldots\nu_l(\omega). \tag{6}$$

Statistic (6) is very "stringent": if it is essentially nonzero (note that $0 \le \kappa(\omega) \le 1$), all components at a given frequency should simultaneously and sufficiently strongly correlate between themselves, and the value of (6) strongly decreases, if only one of its factors is close to zero. This a useful property, because statistic (6) at a given frequency is sensitive only to a general tendency which simultaneously manifests itself in all of the recorded signals. On the other hand, the "resolution" of statistic (6) abruptly decreases even if only one ill-constrained scalar component is incorporated in the vector time series $\mathbf{Z}(t)$ studied.

Therefore, it is desirable to additionally construct, using ν_i, a value that would also describe a general tendency appearing in all signals but would be more stable to an incidental, ill-constrained component included in the set of scalar signals to be analyzed. Such a value may be the arithmetic mean

$$\rho^2(\omega) = \sum_{i=1}^{l} \nu_i^2(\omega)/l. \tag{7}$$

Like (6), $\rho^2(\omega)$ may assume the values from 0 to 1.

DESCRIPTION OF THE CALCULATION METHOD

The calculation of statistics (4), (6), and (7) requires estimation of the spectral matrix $\mathbf{S}_{zz}(\omega)$ of the l-dimensional time series $\mathbf{Z}(t)$. Following [2, 3], we will use the vector autoregression model [9] for the time series $\mathbf{Z}(t)$, characterized by reasonably good frequency resolution for short samples (i.e., relatively narrow time windows).

Let $\mathbf{Z}(t) = (Z_1(t), \ldots, Z_l(t))^T$, $t = 1, \ldots, L$ be a sample from the original multivariate time series in a window of a length of L observations (for definiteness, the first window is assumed). The superscript "T" indicates a transpose.

First, note that the time series analyzed are dominated by low frequencies, since the contribution of low frequencies is commonly much greater than the power of higher-frequency components. Therefore, in order to eliminate the dominating effect of the low-frequency components, the incremental series, rather than $Z_i(t)$, will be considered:

$$\mathbf{z}(t) = \mathbf{Z}(t+1) - \mathbf{Z}(t), \quad t = 1, \ldots, L-1. \tag{8}$$

Furthermore, since the algorithm is designed to the processing of data differing in their physical nature or scale, selective means s_i and variances σ_i of each scalar component $z_i(t)$ are calculated in each time window:

$$s_i = \sum_{t=1}^{L-1} z_i(t)/(L-1),$$
$$\sigma_i^2 = \sum_{t=1}^{L-1} (z_i(t) - s_i)^2/(L-2). \tag{9}$$

Also, each component is normalized to the unit variance:

$$z_i(t) := (z_i(t) - s_i)/\sigma_i, \quad i = 1, \ldots, l. \tag{10}$$

In the method of principal spectral components, in which the eigenvalues of spectral matrices represented the main statistic [1–3], operations (9) and (10) were indispensable to the processing of diverse and multiscale information. As follows from formulas (2) and (3) for the matrices whose eigenvalues (4) are further used, the canonical coherences are invariant under both scaling transformations and any nondegenerate vector filterings of the series $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ [8]. However, operations (9) and (10) are useful in this case as well, since they reduce the effect of the round-off errors related to the finiteness of the numerical format.

Then, the vector autoregression model is constructed for the series $\mathbf{z}(t)$:

$$\mathbf{z}(t) + \sum_{k=1}^{p} \mathbf{A}_k \mathbf{z}(t-k) = \boldsymbol{\zeta}(t), \tag{11}$$

where $p \ge 1$ is the autoregression order, $\mathbf{A}_k$ ($k = 1, \ldots, p$) are the $l \times l$ matrices of the autoregression coefficients, and $\boldsymbol{\zeta}(t)$ is the l-dimensional series of the identification residuals, which is supposed to be a sequence of independent Gaussian vectors with the zero mean and covariation matrix $\mathbf{P}$.

To determine the matrices $\mathbf{P}$ and $\mathbf{A}_k$ ($k = 1, \ldots, p$), the multidimensional procedure of Durbin–Levinson [9] is used, for which sample estimates of the covariation $(l \times l)$-matrices should first be calculated,

$$\mathbf{R}(k) = \langle \mathbf{z}(t+k)\mathbf{z}^H(t) \rangle, \quad k = 0, 1, \ldots, p,$$

where $\langle \ldots \rangle$ means the time averaging. After the matrices $\mathbf{P}$ and $\mathbf{A}_k$ ($k = 1, \ldots, p$) are determined, the complex spectral matrix $\mathbf{S}_{zz}(\omega)$ is estimated by the formula

$$\mathbf{S}_{zz}(\omega) = \mathbf{F}^{-1}(\omega)\mathbf{P}\mathbf{F}^{-H}(\omega), \tag{12}$$

where the complex matrices $\mathbf{F}(\omega)$ are

$$\mathbf{F}(\omega) = \mathbf{I} + \sum_{k=1}^{p} \mathbf{A}_k \exp(-i\omega k), \tag{13}$$

ω is frequency, $\mathbf{I}$ is the unit $(l \times l)$-matrix, and i is the imaginary unit [9]. The matrix $\mathbf{S}_{zz}(\omega)$ being Hermitean

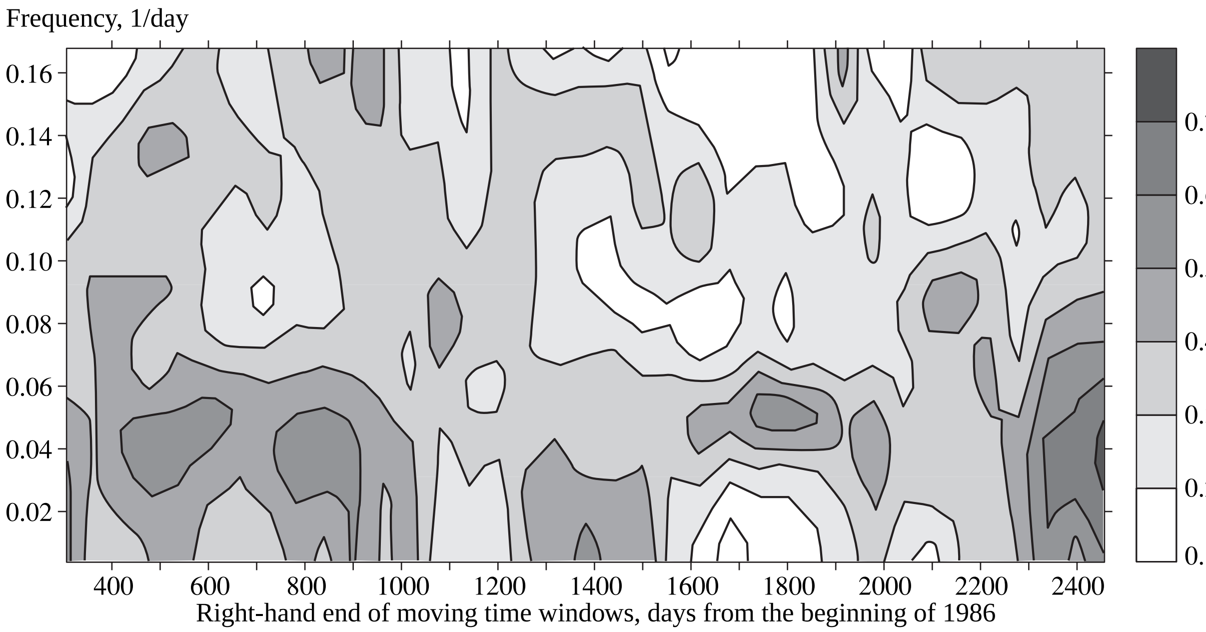

Fig. 1. Evolution of the mean squared values of the component coherences for the four-dimensional time series of the H_4SiO_4 concentration in the groundwater of wells and springs of the Petropavlovsk, Kamchatka research site.

and nonnegative, its eigenvalues are real and nonnegative. Let

$$0 \le \lambda_l(\omega) \le \ldots \le \lambda_2(\omega) \le \lambda_1(\omega)\,, \qquad (14)$$

be eigenvalues of the matrix $\mathbf{S}(\omega)$, arranged in descending order; i.e., λ_1 and λ_l are maximum and minimum eigenvalues, respectively.

The frequency functions $\lambda_i(\omega)$ are calculated within a time window consisting of L observations and moved with a step of ΔL observations ($1 \le \Delta L \le L$). They were analyzed in [1–3]. Spectral matrix (12) being estimated in a moving time window, its eigenvalues depend not only on frequency ω but also on a certain coordinate of the window τ: $\lambda_i = \lambda_i(\tau, \omega)$.

Let Δt be the sampling time interval of monitoring sensors. The parameters τ will be set equal to the coordinate of the right boundary of the time window:

$$\tau = \tau_0 + \Delta t(L - 1 + (j - 1)\Delta L), \qquad (15)$$

where $j = 1, 2, \ldots$ is the window number, and τ_0 is the beginning moment of observations. This choice of τ aims at the use of prediction statistics: the anomaly in their behavior precedes the moment τ.

The variation range of the frequency ω is constrained by the choice of Δt and L:

$$\omega_{\min} = 2\pi/((L-1)\Delta t) \le \omega \le \pi/(\Delta t) = \omega_{\max}. \quad (16)$$

The approach proposed focuses on the analysis of the two-parameter relations

$$\lambda_1(\tau, \omega), \mu_1^2(\tau, \omega), \kappa(\tau, \omega), \rho^2(\tau, \omega) \qquad (17)$$

and the determination of the τ intervals and ω values for which statistics (17) are well above their background values.

Three of statistics (17), namely $\lambda_1(\tau, \omega)$, $\kappa(\tau, \omega)$, and $\rho^2(\tau, \omega)$, are employed to detect an increase in the collective behavior of all components of the multidimensional time series analyzed (note that $\mu_1^2(\tau, \omega)$ is employed for the analysis of the interrelation between two vector series). The question arises as to which statistic is preferable in each specific case. In spite of the "smoothing-out" effect of preliminary operations (8)–(10), the function $\lambda_1(\tau, \omega)$ has a tendency to response to those components of the series $\mathbf{Z}(t)$, which exhibit the highest amplitude variations within the τ interval under study. If some component of $\mathbf{Z}(t)$ has an "abnormal" amplitude anomaly, the latter would inevitably produce a burst in $\lambda_1(\tau, \omega)$ at low frequencies. On the other hand, an anomaly of this kind may be considered as a useful feature, because such visual anomalies are most widespread among possible precursors in the traditional expert analysis of data. Therefore, the multidimensional analysis, designed for the detection of "hidden" anomalies, also recognizes the visible ones.

By their structure, the statistics $\kappa(\tau, \omega)$ and $\rho^2(\tau, \omega)$ depend only on the interrelations between the phases of the $\mathbf{Z}(t)$ components and, for this reason, appear to be preferable in the search for hidden anomalies; however, the available evidence does not confirm decisively this conclusion. For the time being, one may state that all of the three diagrams, namely, $\kappa(\tau, \omega)$, $\rho^2(\tau, \omega)$, and $\lambda_1(\tau, \omega)$, are useful. Often, they provide constraints on the same

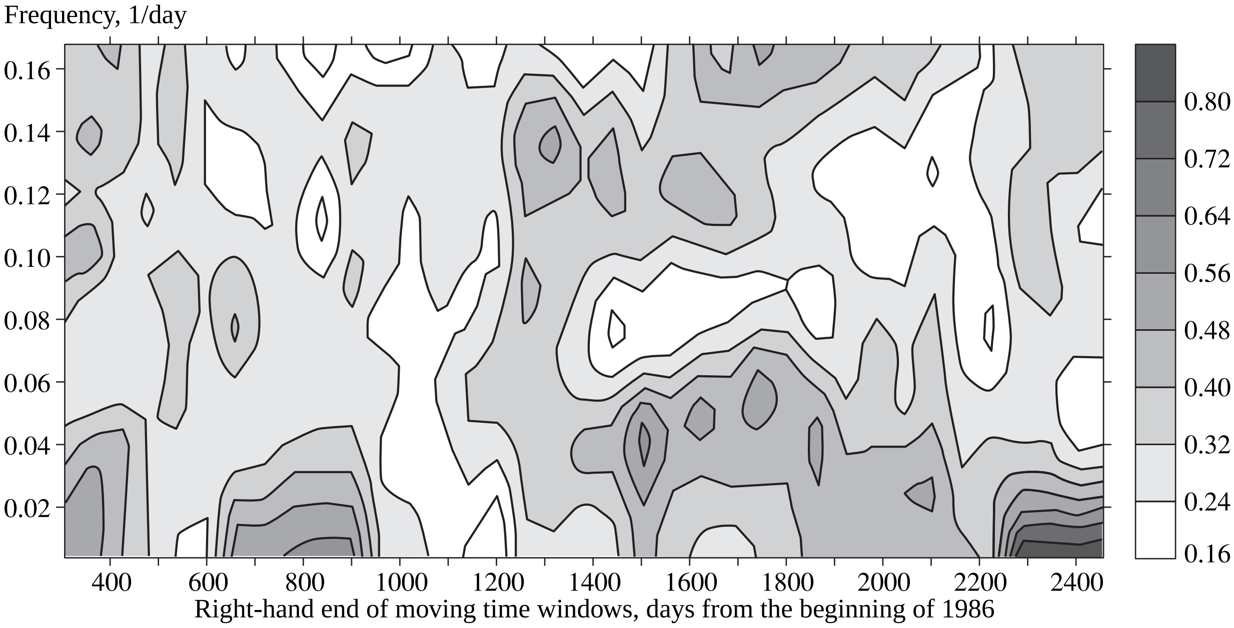

Fig. 2. Evolution of the mean squared values of the component coherences for the five-dimensional time series of the HCO_3^- concentration in the groundwater of wells and springs of the Petropavlovsk, Kamchatka research site.

anomalies (more specifically, the same values of τ and ω). On the other hand, if their results differ, this is an indication that the series $\mathbf{Z}(t)$ may include poorly constrained components whose only contribution is the noise increase (this may be a result, for example, of maloperation of the sensors providing original data).

NUMERICAL EXAMPLE

The numerical experiment was designed to recognize the time intervals and frequency bands in which either the scalar components of the multidimensional time series under study or two groups of such components (for the statistic $\mu_1^2(\tau, \omega)$) vary synchronously. This synchronization can be caused by many factors, which may be classified as follows [2].

(F1) The presence of an external noise source with a large spatial correlation radius, affecting all measurement stations.

(F2) Consolidation of the crustal material in the area covered by the monitoring network.

(F3) Postseismic variations of geophysical fields, produced by strong earthquakes.

The effect of the factor F1 can be strongly reduced through the application of the adaptive procedures of the measured noise reduction [1]. In the context of the search for new precursors of strong earthquakes, the detection of the signal F2 is most interesting, because it is likely to be related to the strong event preparation. In fact, a large amount of energy can be accumulated only in an unbroken crustal zone; otherwise, energy would be dissipated through creep motions and weak shocks. The regularities in the postseismic synchronization (factor F3) of an earthquake sequence may also be useful for prediction purposes. These regularities include, for example, an increase tendency in the number of observation points or types of observed signals, which yield evidence of synchronization.

Lyubushin *et al.* [3] and Lyubushin [2] made a detailed analysis of the diagrams $\lambda_1(\tau, \omega)$ constructed for the time series of variations in the hydrogeochemical properties of a system of wells and overflow springs at the Petropavlovsk, Kamchatka research site. The observations were conducted from the beginning of 1986 through 1992 at a frequency of one observation per three days (data provided by the Experimental Seismological Expedition, Institute of Vulcanology, Far East Division of the Russian Academy of Sciences). The bursts in $\lambda_1(\tau, \omega)$ were compared with seismic events for various combinations of time series and classified according to the factors F2 and F3.

Below, a small part of these data is analyzed in terms of the canonical coherences (Figs. 1–3). All estimates are obtained in the window of a length of $L = 100$ readings (300 days), moving with a step of $\Delta L = 20$ readings (60 days) for the autoregression model (11) of third order ($p = 3$). Each window was preprocessed using operations (8)–(10). Note that the diagrams $\kappa(\tau, \omega)$ were not estimated, because they are very similar to the $\rho^2(\tau, \omega)$ diagrams, differing only in their scales.

The table presents the seismicity data from the vicinity of the research site, a characteristic linear dimension of the observation network being about

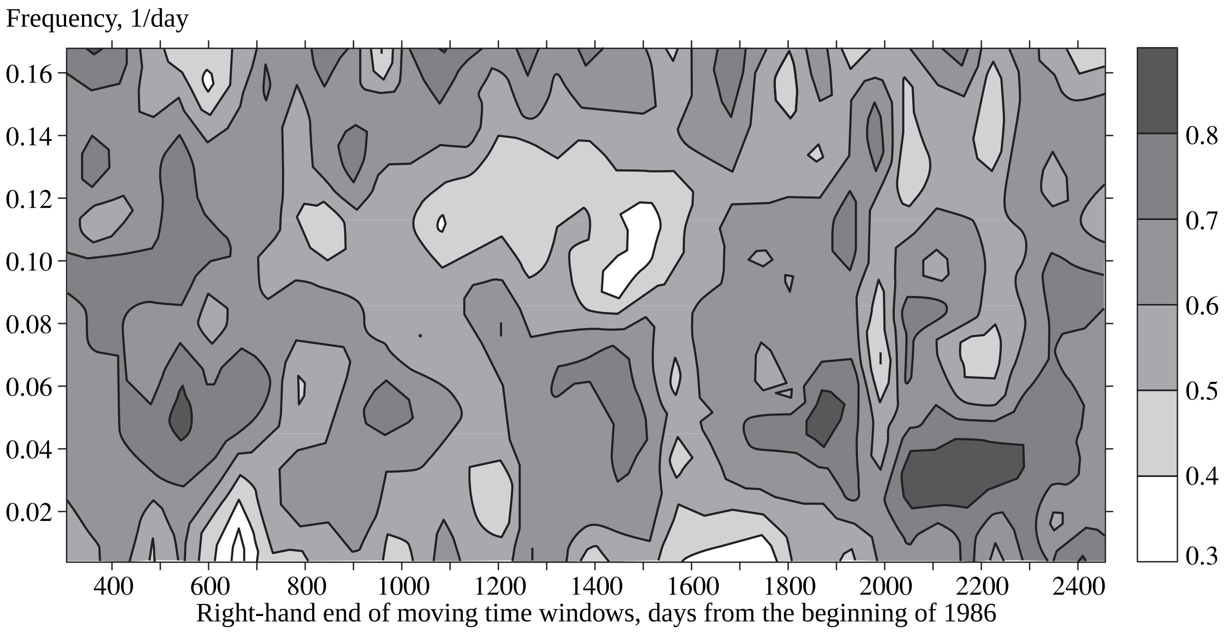

Fig. 3. Evolution of the mean squared absolute values of the maximum canonical coherence between the five-dimensional time series of the HCO_3^- concentration and four-dimensional time series of the H_4SiO_4 concentration in the groundwater of wells and springs of the Petropavlovsk, Kamchatka research site.

50 km. The Pinachevo station is chosen as a center of the site, and its epicentral distances are listed.

Figures 1 and 2 present the $\rho^2(\tau, \omega)$ diagrams of the average evolution of the squared component coherences for the four-dimensional series of the H_4SiO_4 concentration and five-dimensional series of the HCO_3^-, respectively. The comparison of these results with the seismicity data from table shows no pronounced synchronization precursor signal for the strongest earthquake no. 7; both series response to this event by obviously postseismic synchronization (factor F3).

Main evidence on the earthquakes of 1986–1992 (after the ESE IV data [3, 4]) that produced changes in the behavior of the observation springs and wells at the Petropavlovsk research site

No.	Date	Number of days after the beginning of 1986	Distance to the Pinachevo station	Magnitude	Depth, km
1	Jun. 17, 1986	168	160	5.0	40
2	Oct. 6, 1987	644	130	6.6	34
3	Sep. 15, 1989	1354	105	4.9	44
4	Mar. 1, 1990	1521	120	5.8	24
5	Dec. 19, 1990	1814	155	6.1	24
6	Apr. 8, 1991	1924	170	4.7	139
7	Mar. 2, 1992	2253	115	7.1	40

The synchronization precedes event no. 2 (next in magnitude), but this signal is rather weak.

Figure 3 shows the dependence $\mu_1^2(\tau, \omega)$, i.e., the evolution of the squared absolute value of the maximum canonical coherence between the five-dimensional series of the HCO_3^- concentration and four-dimensional series of the H_4SiO_4 concentration ($n = 4$, $m = 5$, $l = n + m = 9$). The most pronounced and significant anomaly in μ_1^2 is observed at $\tau \cong 2000–2280$ days within the frequency band $\omega \cong 0.02–0.06$ 1/day and obviously precedes the strongest earthquake no. 7 with $M = 7.1$ and $t = 2253$ days (see table). As is mentioned above, none of the separate ρ^2 estimates for the series HCO_3^- and H_4SiO_4 (signals F2) yields such a significant precursor (see Figs. 1 and 2). This is a characteristic example of a "hidden" signal which is involved in the interaction between processes rather than in each process separately. Note also that, as is seen from Fig. 3, event no. 2 (next in magnitude) is preceded by a precursor in the form of a burst at $\omega \cong 0.05$ 1/day, although it is not so prominent as the burst preceding event no. 7.

CONCLUSION

An approach is offered for the synchronization signal detection in the variations of diverse scalar components of multidimensional time series measured by geophysical monitoring systems. This approach, which is based on the canonical coherences estimated in a moving time window, is an alternative to the analysis of the maximum eigenvalue of the spectral matrix, previously proposed by the author. A feature, specific to the method offered here, is its application to the frequency–time analysis of the interrelation between two geophysical fields recorded by a network of observation stations.

A numerical example of the processing of multidimensional geochemical observation series from a seismically active region (Kamchatka) demonstrates that the method is capable of detecting a hidden precursor recognizable only from the consideration of the interrelation between two geochemical fields.

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