

Comparison of Bayesian estimates of peak ground acceleration (A_{max}) with PSHA in Iran

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Abstract

Bayesian probability theory is an appropriate and useful method for estimating parameters in seismic hazard analysis. The analysis in Bayesian approaches is based on *a posterior* belief, also their special ability is to take into account the uncertainty of parameters in probabilistic relations and *a priori* knowledge. In this study, we benefited the Bayesian approach in order to estimate maximum values of peak ground acceleration (A_{max}) also quantiles of the relevant probabilistic distributions are figured out in a desired future interval time in Iran. The main assumptions are Poissonian character of the seismic events flow and properties of the Gutenberg-Richter distribution law. The map of maximum possible values of A_{max} and also map of 90% quantile of distribution of maximum values of A_{max} on a future interval time 100 years is presented. According to the results, the maximum value of the A_{max} is estimated for Bandar-Abbas as 0.3g and the minimum one is attributed to Esfahan as 0.03g. Finally, the estimated values in Bayesian approach are compared with what was presented applying probabilistic seismic hazard (PSH) methods based on Cornell method carried out in Iran. The distribution function of A_{max} for future time intervals of 100 and 475 years are calculated for confidence limit of probability level of 90%.

Key words: Seismic hazard, Bayesian approach, PGA, PSH method, Iran

1. Introduction

The Iranian plateau is a relatively wide zone of compressional deformation along the Alpine-Himalayan active mountain belt, bounded in the South by the Arabian plate and in the North by the Eurasian plate. The Iranian plateau is comprised of five main features, namely the Zagros Mountains, the Kopeh Dagh, the Makran complex, the Alborz-Azerbaijani and the central Iranian block. According to the five features, five Seismotectonic province intended for Iran by Mirzaei et al. (1998). Seismicity map of Iran, including seismotectonic provinces is demonstrated in Fig. 1.

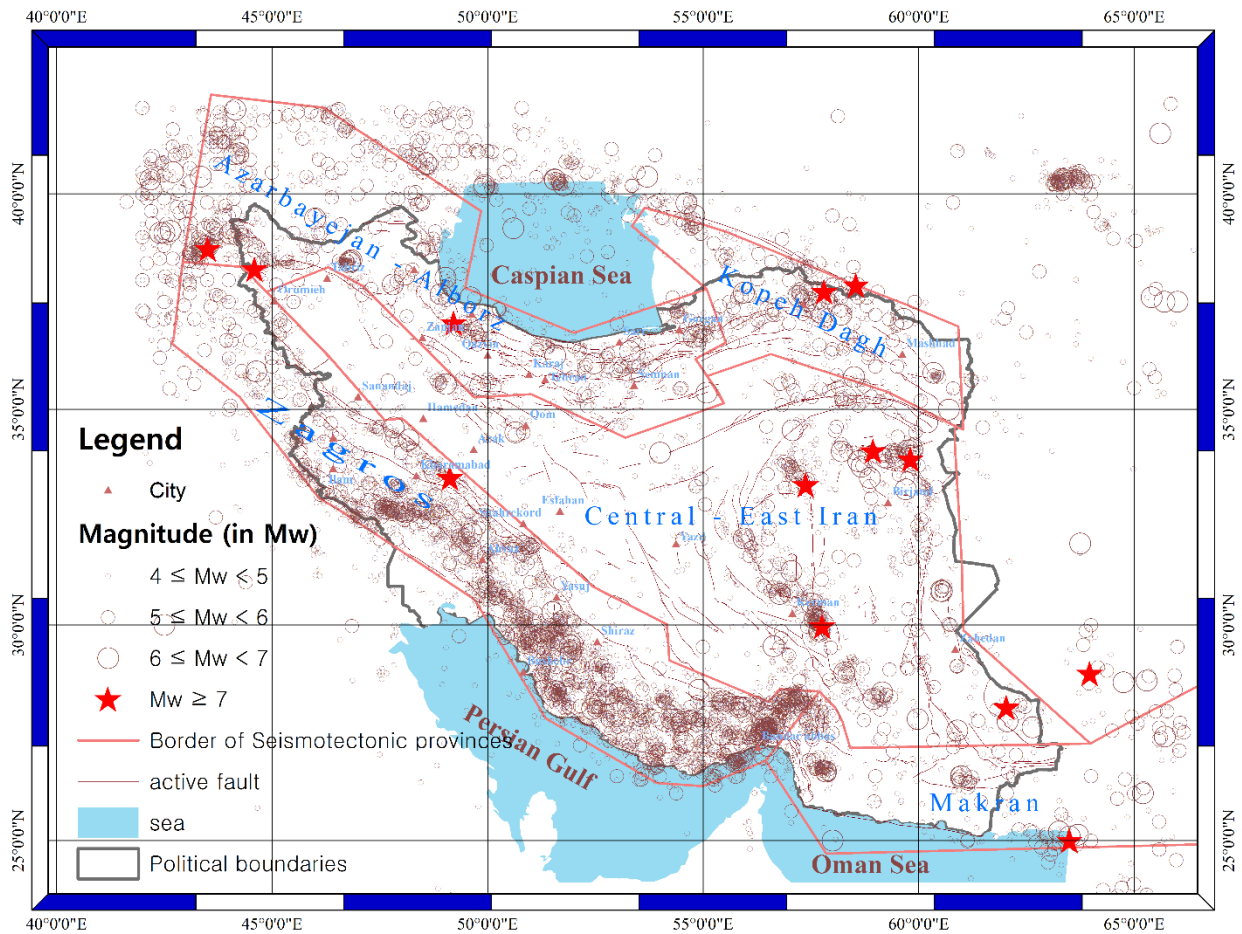


Fig 1. Seismicity map of Iran including seismotectonic provinces based on Mizaei et al. (1998) and locations of earthquakes larger than Mw 4

Iran and its surrounding region have experienced repeated moderate to large magnitude earthquakes during the previous centuries. Not being able to predict the occurrence of earthquakes, seismic hazard analysis is a practical approach for engineering seismology to estimate ground motions at a given site. Different people with different methods have tried to estimate seismic hazard in Iran; Tavakoli and Ghafory-Ashtiany (1999), Moinfar et al. (2000), Ghodrati Amiri et al. (2008), Vafaie et al. (2011), Moinfar et al. (2012), Mousavi Bafrouei et al. (2014), Boostan et al. (2015), Khodaverdian et al. (2016) and Khoshnevis et al. (2017).

Bayesian techniques provide a rigorous means of combining prior information on seismicity whether it is judgmental, geological or statistical with historical observations of earthquake occurrences (Galanis et al. 2002) and ready framework for the propagation of uncertainty through the risk models is supplied with probability distribution which is represented through Bayesian approach (Kelly and Smith 2011). In this study, using the above mentioned characteristics of the Bayesian method, the benefits of combining judgmental information about hazard parameters are examined in seismic hazard analysis. Bayesian approach also provides conditions that we can insert uncertainty in our calculation. The present Bayesian approach was elaborated in the works of Pisarenko et al. (1996), Pisarenko and Lyubushin (1997, 1999). Later, Lyubushin and Parvez (2010) modified creating maps of Bayesian estimates of peak ground acceleration statistics. The main computational code of the method which was elaborated by Lyubushin, has been applied to estimate seismic hazard in different regions of the world (Ruzhich et al. 1998; Lyubushin et al. 2002; Tsapanos et al. 2001; Tsapanos 2003; Tsapanos and Christova 2003; Yadav et al. 2012, 2013; Bayrak and Turker 2016, 2017; Mohammadi et al. 2016). This study deals with investigating the maximum peak ground accelerations (PGA) which makes a significant difference with the above mentioned studies, so that the basic method has gone through strong modifications.

Due to the high seismicity of Iran, researchers are always seeking to apply the latest methods to provide seismic hazard analysis in Iran. For this reason, in this study, the method proposed by Pisarenko et al. (1996) is used for the first time to estimate PGA values in Iran and compare it with the results of the modified probabilistic method previously performed by Mousavi Bafrouei et al. (2014). Since the applied attenuation relation is very effective in the resultant PGA values, equal relations are exerted to make a fair comparison of the two methods.

2. Input data

The method proposed by Pisarenko et al (1996), in addition to estimating seismicity parameters, it is also helpful to find PGAs. The catalog of earthquakes is the most important prerequisite in this method. In this regard, for this study the seismic catalog of Mousavi Bafrouei et al. (2015) is made updated by mid of 2017 by referring to USGS and ISC. The total number of instrumental events recorded by mid 2017 which used in this study will reach over 12,000 events. On the other hand, there are 258 historical events in the catalog, the oldest one date back to 400 BC. In order to estimate PGA, all available data are used to examine the sensitivity of the method proposed by Pisarenko et al. (1996). All data in this study are unified to the M_w scale (Fig. 1). To convert the scale of events from the magnitudes of m_b or M_s reported by ISC or USGS, the relationships provided by Mousavi Bafrouei et al. (2015) have been used. One of the most important assumptions used in the Pisarenko et al. (1996) method is the Poisson character of events. So we only need the major events, and the associated events (i.e. aftershocks) are eliminated from the total data. For this purpose, we have used Gardner and Knopoff (1974) method, of course, some modifications to this method, such as Lyubushin and Parvez (2010), as well as Lyubushin et al. (2002). The complete details of how to remove the dependent processes are fully presented in Lyubushin et al. (2002).

The second input in this approach is the selection of the appropriate attenuation law. Shoja-Taheri et al. (2010) have evaluated the efficiency of some New Generation Attenuation (NGA) models if they are proper to be used in Iran. They proposed Boore and Atkinson (2008), Campbell and Bozorgnia (2008) and Chiou and Youngs (2008) attenuation models can be applied properly in Iran. In this regard, in this study, these three NGAs are applied with the same weight.

3. The Method applied

Let R be some value, which was measured or estimated as a sequence on a “past” time interval $(-\tau, 0)$:

$$\bar{R}^{(n)} = (R_1, \dots, R_n), R_i \geq R_0, R_\tau, R_\tau = \max_{1 \leq i \leq n} (R_1, \dots, R_n) \quad (1)$$

Values (1) could have an arbitrary physical nature. Below we shall consider (1) as magnitudes of seismic events in some regions or logarithm of seismic peak ground accelerations at a given site. R_0 is a minimal cutoff value, i.e. such value which is defined by possibilities of registration systems or was chosen as minimal value up from which values sequence (1) is statistically representative.

First our assumption is that values (1) obey the Gutenberg-Richter law of distribution:

$$\Pr\{R < x\} = F(x|R_0, \rho, \beta) = \frac{e^{-\beta.R_0} - e^{-\beta.x}}{e^{-\beta.R_0} - e^{-\beta.\rho}}, R_0 \leq x \leq \rho \quad (2)$$

Here ρ is the unknown parameter that has a sense of maximal possible value of R . Unknown parameter β usually is called as “slope” of the Gutenberg-Richter when the dependence (2) is plotted in doubly logarithmic axes.

Our second assumption is that the sequence (1) is a Poissonian process with some intensity value λ , which is unknown parameter also. Thus, the full vector of unknown parameter is the following:

$$\theta = (\rho, \beta, \lambda) \quad (3)$$

For brevity all functions of distribution and statistics of the sequence (1) we shall denote as $\cdot(\cdot | \theta)$, for example, (2) - as $F(x|\theta)$, argument R_0 will be omitted.

Probabilistic density of distribution, according to the law (2):

$$f(x|\theta) = F'(x|\theta) = \frac{\beta \cdot e^{-\beta \cdot x}}{e^{-\beta \cdot R_0} - e^{-\beta \cdot \rho}} \quad (4)$$

Let's introduce now an error ε , with which we know values (1), i.e. for us really in (1) are accessible not *true*, but *apparent* values of R , which are defined by formula:

$$\tilde{R} = R + \varepsilon \quad (5)$$

and let $n(x|\delta)$ be a density of probabilistic distribution of the error ε , where δ is a given scale parameter of the density. We shall use below a uniform distribution density:

$$n(x|\delta) = \begin{cases} 1/2\delta, & |x| \leq \delta \\ 0, & |x| > \delta \end{cases} \quad (6)$$

Then a distribution density of the apparent values is the following:

$$\tilde{f}(x|\theta, \delta) = \int_{-\infty}^{+\infty} f'(\xi|\theta)n(x - \xi|\delta)d\xi = \frac{F(x+\delta|\theta) - F(x-\delta|\theta)}{2\delta} \quad (7)$$

Let $\tilde{F}(x|\theta, \delta)$ be a function of distribution which is corresponding to the density (7). Because $F(x|\theta) = 0$ for $x < R_0$, then

$$\tilde{F}(x|\theta, \delta) = \int_{R_0 - \delta}^x \tilde{f}(\xi|\theta, \delta)d\xi \quad (8)$$

As apparent values $\tilde{R} \geq R_0$ and $\tilde{f}(x|\theta, \delta) > 0$ for $x \in (R_0 - \delta, R_0)$ then we shall renormalize $\tilde{f}(x|\theta, \delta)$ and $\tilde{F}(x|\theta, \delta)$ in such a way that they will equal zero for $x < R_0$:

$$\bar{f}(x|\theta, \delta) = \begin{cases} \frac{\tilde{f}(x|\theta, \delta)}{1 - \tilde{F}(R_0 | \theta, \delta)}, & x \geq R_0 \\ 0, & x < R_0 \end{cases} \quad (9)$$

Function of distribution, which corresponds to the density (9), is defined by the formula:

$$\bar{F}(x|\theta, \delta) = \begin{cases} \frac{\bar{F}(x|\theta, \delta) - \bar{F}(R_0|\theta, \delta)}{1 - \bar{F}(R_0|\theta, \delta)} & , \quad x \geq R_0 \\ 0 & , \quad x < R_0 \end{cases} \quad (10)$$

Now we want to derive a relationship between intensity λ of “true” R-values and intensity $\bar{\lambda}$ of their apparent values. As $R = \tilde{R} - \varepsilon$ and $-\varepsilon$ is distributed according to (6) also, then

$$f(x|\theta, \delta) = \int_{-\infty}^{+\infty} \bar{f}(\xi|\theta, \delta) \cdot n(x - \xi|\delta) d\xi \quad (11)$$

A share of those apparent R-values, for which true values $< R_0$ equals:

$$\kappa = \int_{-\infty}^{R_0} f(x|\theta) dx = \int_{-\infty}^{R_0} \int_{-\infty}^{+\infty} \bar{f}(\xi|\theta, \delta) \cdot n(x - \xi|\delta) d\xi dx \quad (12)$$

Then, in the assumption of the Poissonian character of the sequence (1) it follows that:

$$\lambda = \bar{\lambda} \cdot (1 - \kappa) \quad (13)$$

Substituting (7) into (12) and using the fact that $\bar{F} = 0$ for $x < R_0$ we'll obtain:

$$\kappa = \int_{-\infty}^{R_0} \frac{\bar{F}(x + \delta|\theta, \delta) - \bar{F}(x - \delta|\theta, \delta)}{2\delta} dx = \frac{1}{2\delta} \int_{R_0 - \delta}^{R_0} \bar{F}(x + \delta|\theta, \delta) dx = \frac{1}{2\delta} \int_{R_0}^{R_0 + \delta} \bar{F}(x|\theta, \delta) dx$$

Thus:

$$\bar{\lambda} = \bar{\lambda}(\theta, \delta) = \frac{\lambda}{1 - \frac{1}{2\delta} \int_{R_0}^{R_0 + \delta} \bar{F}(x|\theta, \delta) dx} \quad (14)$$

Let Π be *a priori* uncertainty domain of values of parameters θ :

$$\Pi = \{\lambda_{\min} \leq \lambda \leq \lambda_{\max}, \beta_{\min} \leq \beta \leq \beta_{\max}, \rho_{\min} \leq \rho \leq \rho_{\max}\} \quad (15)$$

We shall consider *a priori* density of the vector θ to be uniform in the domain Π .

Let $[0, T]$ be a future interval of time for which we want to estimate function of distribution of maximal value ρ and its quantiles. As the flow of events (1) is stationary and Poissonian then it is follows that intensity of event with $R < x$ equals $\lambda \cdot F(x|\theta)$ and intensity of events with $R \geq x$

equals $\lambda \cdot (1 - F(x|\theta))$. From Poissonian character of the sequence (1) it follows that probability that it will be no events with $R \geq x$ on time interval $[0, T]$ or that all events on $[0, T]$ will have $R < x$ equals:

$$\exp(-\lambda \cdot (1 - F(x|\theta)) \cdot T) \quad (16)$$

Let's denote by R_T maximal value of R on the time interval $[0, T]$. Then $\Pr\{R_T < x\} = \exp(-\lambda \cdot (1 - F(x|\theta)) \cdot T)$. But into this probability a case when there are no events on $[0, T]$ is included also. Let's denote by ν_T the number of events with $R \geq R_0$ on the interval $[0, T]$. Then

$$\Pr\{\nu_T = 0\} = e^{-\lambda \cdot T} \quad \Pr\{\nu_T \geq 1\} = 1 - e^{-\lambda \cdot T}$$

That is why:

$$\Phi_T(x|\theta) = \Pr\{R_T < x | \nu_T \geq 1\} = \frac{\exp(-\lambda T(1-F(x|\theta))) - \exp(-\lambda T)}{1 - \exp(-\lambda T)} = \frac{\exp(\lambda T F(x|\theta)) - 1}{\exp(\lambda T) - 1} \quad (17)$$

Formula (17) defines an expression for *a priori* function of distribution for *true* maximal values of R on time interval $[0, T]$. Let's introduce also the following functions:

$$\phi_T(x|\theta) = \frac{d}{dx} \Phi_T(x|\theta) \quad (18)$$

- *a priori* density for true maximal values of R on time interval $[0, T]$;

$$Y_T(\alpha|\theta) \text{- a root of equation: } \Phi_T(x|\theta) = \alpha, 0 \leq \alpha \leq 1 \quad (19)$$

- a priory quantile for probability α for true maximal values of R on time interval $[0, T]$;

If we substitute in formula (17) $F(x|\theta) \longrightarrow \bar{F}(x|\theta, \delta)$ then we'll obtain a function:

$\Phi_T(x|\theta, \delta)$ - *a priori* function of distribution for apparent maximal values of R on time interval $[0, T]$.

Substituting

$\bar{\Phi}_T(x|\theta, \delta)$ into formulae (18) and (19), we'll obtain:

$\bar{\Phi}_T(x|\theta, \delta)$ - *a priori* density for apparent maximal values of R on time interval [0,T] and:

$\bar{Y}_T(\alpha|\theta, \delta)$ - *a priori* quantile for probability α for apparent maximal values of R on time interval[0,T].

According to definition of conditional probability, *a posterior* density of distribution of vector of parameters θ equals to:

$$f(\theta|\bar{R}^{(n)}, \delta) = \frac{f(\theta, \bar{R}^{(n)}|\delta)}{f(\bar{R}^{(n)}|\delta)} \quad (20)$$

but $f(\theta, \bar{R}^{(n)}|\delta) = f(\bar{R}^{(n)}|\theta, \delta) \cdot f^a(\theta)$, where $f^a(\theta)$ is a priory density of distribution of vector θ in the domain Π . As $f^a(\theta) = \text{const}$ according to our assumption and taking into consideration that:

$$f(\bar{R}^{(n)}|\delta) = \int_{\Pi} f(\bar{R}^{(n)}|\theta, \delta) d\theta$$

we'll obtain after using a Bayes formula [Rao, 1965] and normalizing the density that:

$$f(\theta|\bar{R}^{(n)}, \delta) = \frac{f(\bar{R}^{(n)}|\theta, \delta)}{\int_{\Pi} f(\bar{R}^{(n)}|\vartheta, \delta) d\vartheta} \quad (21)$$

Formula (21) is our main formula for computing *a posterior* density of distribution of vector of parameters θ . In order to use (21) we must have an expression for the function $f(\bar{R}^{(n)}|\theta, \delta)$. Having the assumption of Poissonian character of the sequence (1) and of independency of its members, we can obtain:

$$f(\bar{R}^{(n)}|\theta, \delta) = \bar{f}(R_1|\theta, \delta) \dots \bar{f}(R_n|\theta, \delta) \cdot \frac{\exp(-\bar{\lambda}(\theta, \delta) \cdot \tau) \cdot (\bar{\lambda}(\theta, \delta) \cdot \tau)^n}{n!} \quad (22)$$

Now we are ready completely to compute a Bayesian estimate of vector θ :

$$\hat{\theta}(\vec{R}^{(n)}|\delta) = \int_{\Pi} \vartheta. f(\vartheta|\vec{R}^{(n)}, \delta) d\vartheta. \quad (23)$$

Among one of its component vector (23) contains an estimate of maximum value ρ . Using analogous to (23) formulae, we can obtain Bayesian estimates of any of the functions (17), (18), (19). The most interesting for us are estimates of quantiles of functions of distribution of true and apparent R-values on a given future time interval $[0, T]$, for instance for α -quantiles of apparent values:

$$\hat{Y}_T(\alpha|\vec{R}^{(n)}, \delta) = \int_{\Pi} \bar{Y}_T(\alpha|\vartheta, \delta). f(\vartheta|\vec{R}^{(n)}, \delta) d\vartheta \quad (24)$$

$\hat{Y}_T(\alpha|\vec{R}^{(n)}, \delta)$ for α -quantiles of true values is written analogously to (24). Using averaging over the density (20), (21) we can estimate also variances of Bayesian estimates (23), (24). For example

$$\text{Var}\{\hat{Y}_T(\alpha|\vec{R}^{(n)}, \delta)\} = \int_{\Pi} (\bar{Y}_T(\alpha|\vartheta, \delta) - \hat{Y}_T(\alpha|\vec{R}^{(n)}, \delta))^2. f(\vartheta|\vec{R}^{(n)}, \delta) d\vartheta \quad (25)$$

In order to finish description of the method, we must define the domain of *a priori* uncertainty Π (15).

First of all we set $\rho_{min} = R_{\tau} - \delta$. As for value of ρ_{max} , it is introduced by the user of the method and depends of the specifics of the data series (1). Boundary values for the slope β are defined by formulae:

$$\beta_{min} = \beta_0 \cdot (1 - \gamma), \beta_{max} = \beta_0 \cdot (1 + \gamma), 0 < \gamma \leq 1 \quad (26)$$

Where β_0 is the “central” value, obtained as a maximum likelihood estimate of the slope for Gutenberg-Richter law:

$$\sum_{i=1}^n \ln \left\{ \frac{\beta \cdot e^{-\beta \cdot R_{\tau}}}{e^{-\beta \cdot R_0} - e^{-\beta \cdot R_{\tau}}} \right\} \rightarrow \max_{\beta, \beta \in (0, \beta_{\delta})} \quad (27)$$

Here β_{δ} is a rather big value, for example 10, value γ is a parameter of the method, usually we take

$$\gamma = 0.5.$$

For setting boundary values for intensity in (15) we use the following reasons. As a consequence of normal approximation for Poissonian process for rather big n [Cox, Lewis, 1966], variance of the value λ_τ has approximate value $\sqrt{n} \approx \sqrt{\lambda\tau}$. So taking boundaries $\pm 3\sigma$, we'll obtain:

$$\lambda_{\min} = \lambda_0 \cdot \left(1 - \frac{3}{\sqrt{\lambda_0\tau}}\right), \lambda_{\max} = \lambda_0 \cdot \left(1 + \frac{3}{\sqrt{\lambda_0\tau}}\right) \quad (28)$$

where $\lambda_0 = \frac{\bar{\lambda}_0}{c_f(\beta_0, \delta)}$, $\bar{\lambda}_0 = \frac{n}{\tau}$.

It is worth mentioning that this method is based on the statistical method introduced by Cornell (1968), Benjamin and Cornell (1970). One means of differentiations of this method is the arranging type of earthquake sources and calculation of the source parameters. In this method, for each point of the grid, the corresponding parameters (ρ, β, λ) are calculated using a sequence including logarithm of acceleration values from adjacent events. β -value and λ -value are exactly the same concept of the similar values of seismicity parameters, with a difference that these values are obtained according to the acceleration values from the catalog and the attenuation relations. In other words, magnitude values are substituted by the logarithm of peak ground acceleration values which are deduced from substituting magnitude and distance in attenuation relations or NGAs. In this method, in order to obtain PGA, no regional sources based on seismotectonic studies are not applied. This can be very useful due to the lack knowledge of fault geometries.

We have estimated the parameter ρ - maximum possible value of A_{\max} and a quantile of probability $\alpha = 0.90$ for a future time interval of the length $T=100$ years in a grid of the size 200×200 nodes by latitude and longitude within rectangular $25^\circ \leq \text{Lat} \leq 40^\circ$; $44^\circ \leq \text{Lon} \leq 64^\circ$. The estimates were performed in the following way: for each node of the grid a sequence of A_{\max} was computed using seismic catalog and NGAs in Iran. The next step was removing aftershocks, as it is described in Lyubushin et al. (2002), in order to provide random character of time moments sequence. After that, the only 30

“main-shocks” events which have the maximum values of A_{\max} were taken for the analysis. Thus, for each node of the grid we have the same value of $n=30$ in the relation (1) but different values of R_0 and $R_\tau = \max_{1 \leq i \leq n} R_i$ the *a priori* boundary value for ρ was taken as $\rho_{\max} = R_\tau + 0.5$.

4. Result and discussion

Having estimates of different Bayesian statistics within nodes of regular grid it is possible to create their maps. For reducing the errors these maps were plotted after smoothing the corresponding grid-values of Gaussian kernel functions with radius 1 degree using 100 nearest neighbors' values (Hardle 1989). For the territory of Iran for each attenuation law which was previously introduced we have created maps of the maximum logarithm of peak ground acceleration A_{\max} and 90%-quantile of distribution of A_{\max} at the future time interval of the length 100 years and calculated mean map from these three maps.

One of the main conditions to be investigated in this method is the similarity of the empirical tail probability function with the Gutenberg-Richter law. The graphs for 6 nodes of the grid are demonstrated in Fig. 2.

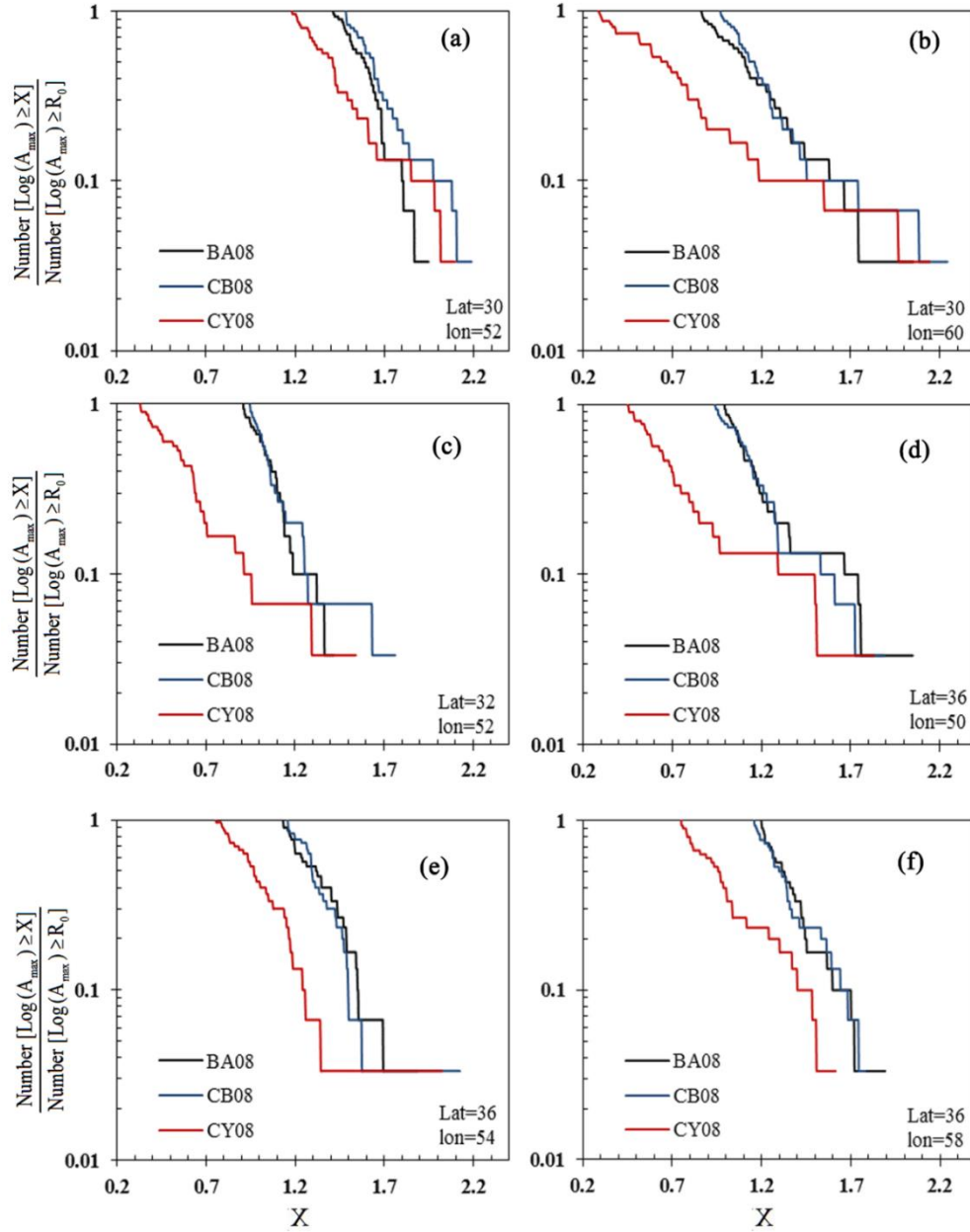


Fig 2. Empirical tail functions of distribution of $\log(A_{\max})$ values, computed for six nodes in the grid for three attenuation laws; BA08 (Boore and Atkinson 2008), CB08 (Campbell and Bozorgnia 2008) and CY08 (Chiou and Youngs 2008)), the horizontal axis (X) is in logarithmic scale and it is cm/s^2 .

It is to be noted that 30 events were considered for each node. When moderate events, such as 30 events, for each node are selected, usually the resultant graphs (Fig. 2) are closer to the Gutenberg-Richter law. Therefore, choosing wrong R_0 values makes our results will go far away from reality.

Hazard map for maximum value of A_{max} by using the 3 attenuation law with the same weight and seismic catalog demonstrate in Fig. 3.

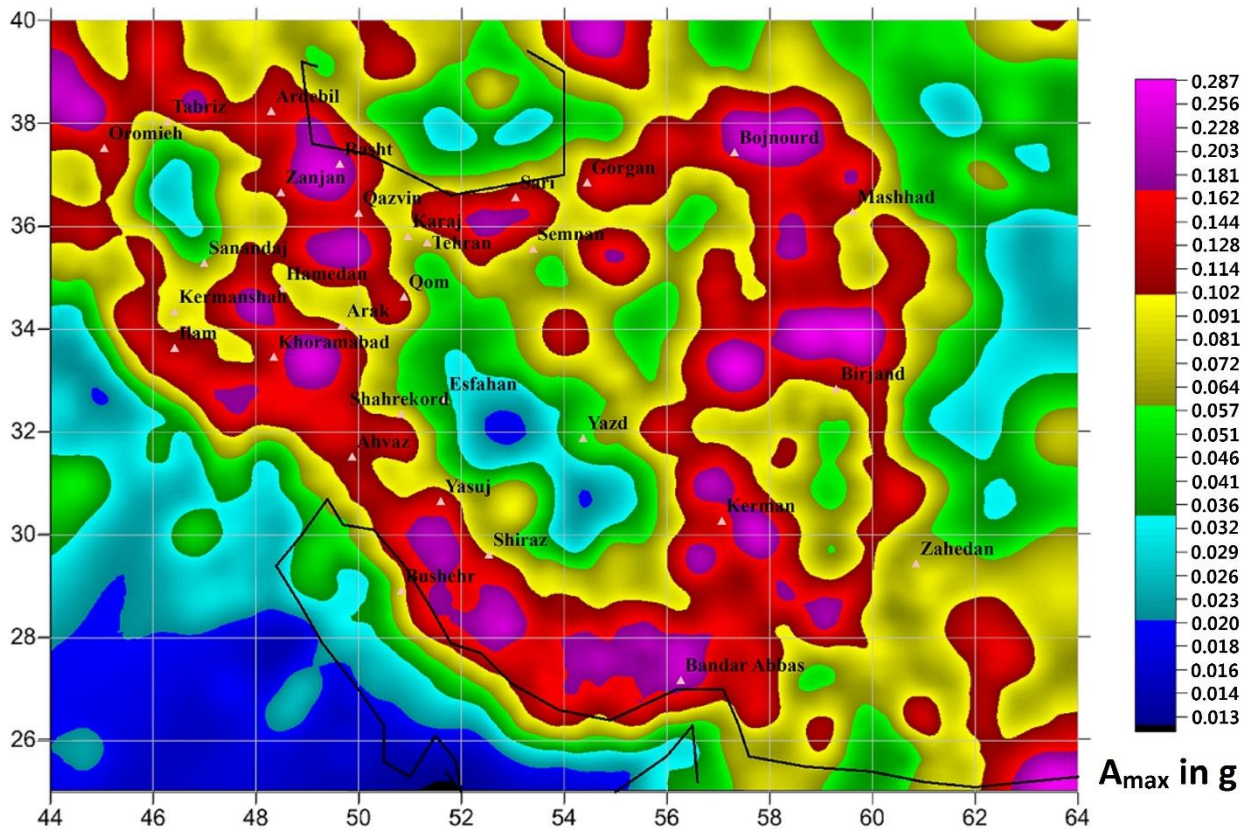


Fig 3. The map of A_{max} in g, for mean of three attenuation laws of Boore and Atkinson (2008), Campbell and Bozorgnia (2008) and Chiou and Youngs (2008). The values are estimated from the Equation 23 and are relevant to the ρ parameter.

For earthquake engineers, hazard maps at different levels of probability are much more important and more practical. In this approach in addition to calculation of value of A_{max} , calculation its quantile of distribution functions. Fig. 4, demonstrate the results of these calculations. This map is 90% quantile of distribution of maximum values of A_{max} on the future time interval of the length $T=100$ years.

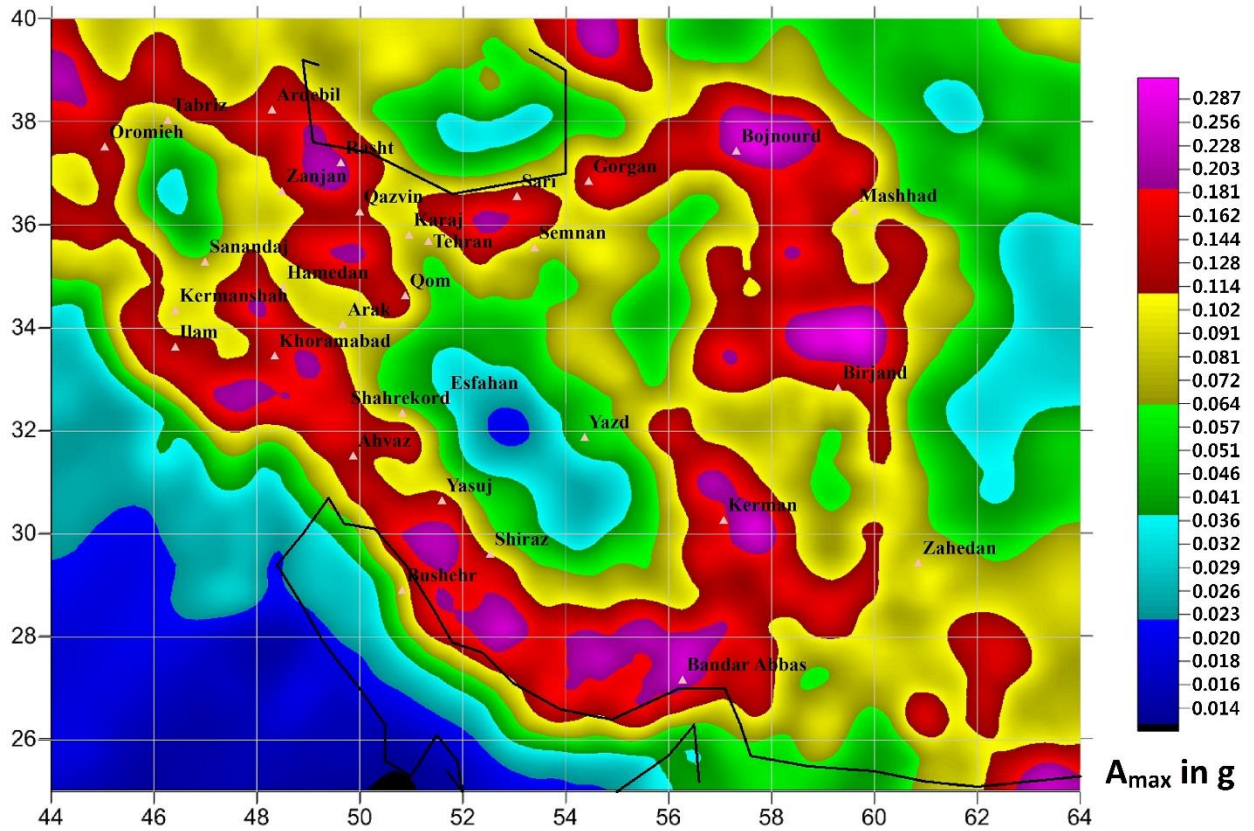


Fig 4. The map of 90% quantile of distribution of A_{max} in g in the future time interval $T=100$ years for mean of the three attenuation laws; Boore and Atkinson (2008), Campbell and Bozorgnia (2008) and Chiou and Youngs (2008). The values are estimated from the Equation 24 and related to the ρ parameter for $\alpha = 0.9$.

4.1 Comparison with PSHA results in Iran

According to Figs. 1 and 5, there is a lack of data in many parts of Iran also incomplete historical data is evident in different parts. Therefore, computing seismicity parameters in practice will not be straightforward. To handle this problem in low seismic zones Mousavi Bafrouei et al. (2014) applied modified PSHA based on what Shi et al. (1992) suggested. In this study, we applied the method proposed by Pisarenko et al. (1996) for Iran data and we will compare results with what Mousavi Bafrouei et al. (2014) deduced in eight cities with different abundances of historical and instrumental data. Names and locations of studied cities can be seen on the map in Fig. 5.

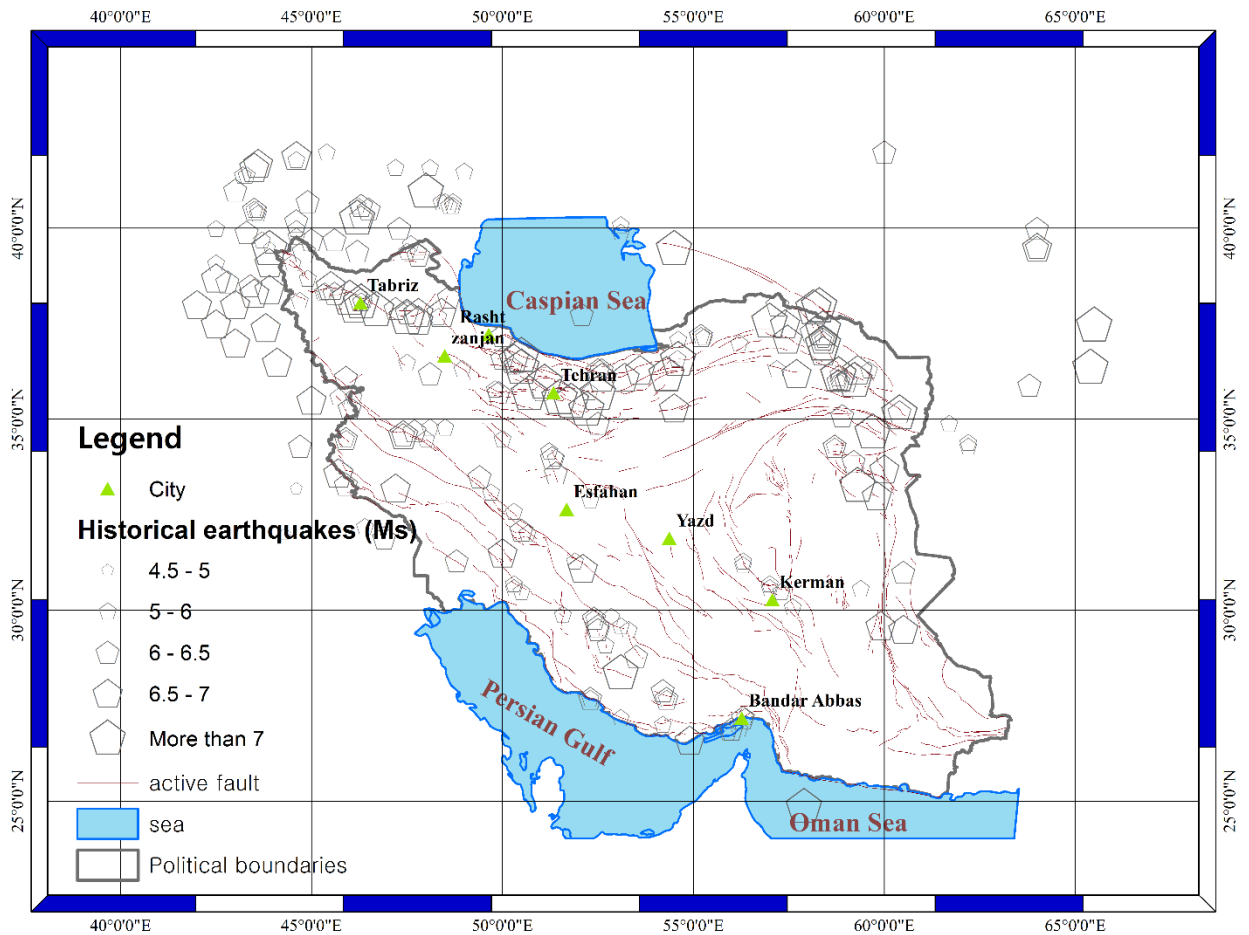


Fig 5. Fault map of Iran including locations of historical earthquakes according to the earthquake catalog of Mousavi Bafrouei et al. (2015) and locations of eight large cities to compare the estimated A_{max} .

According to Figs. 1 and 5 it is clear that in cities of Esfahan and Yazd, in spite of being located to active faults, they have not experienced large earthquakes. Instead in Tabriz and Tehran, large historical and instrumental earthquakes are experienced and high levels of earth ground motion accelerations are recorded.

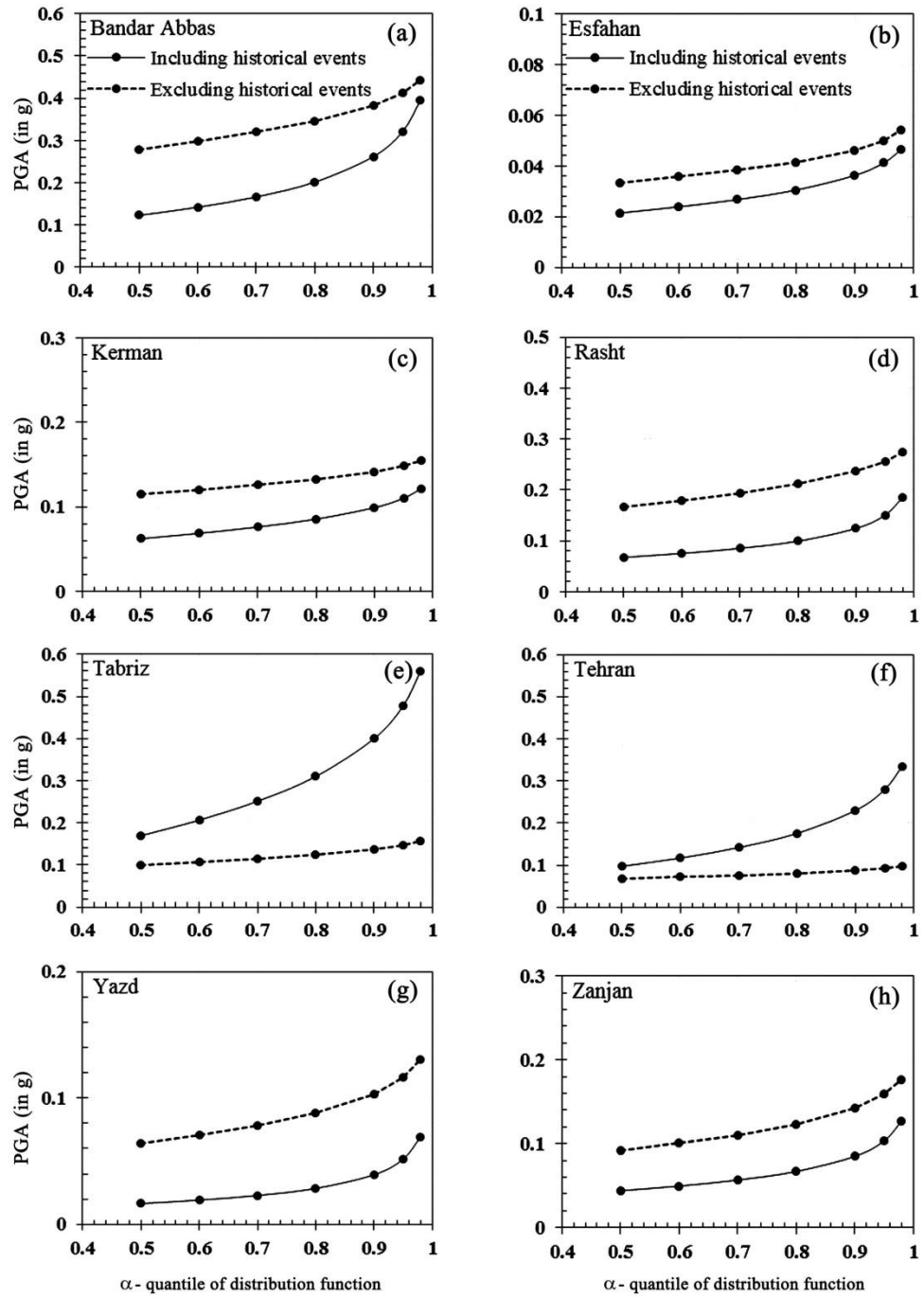


Fig 6. Diagrams of quantiles of distribution function of A_{\max} for future 475 years, in eight cities of Iran in g, for two cases of including and excluding historical data applying the mean of three attenuation laws; Boore and Atkinson (2008), Campbell and Bozorgnia (2008) and Chiou and Youngs (2008). The values correspond to 90% quantile presented in Table 1.

PGAs calculated in method of Pisarenko et al. (1996) are presented as quantiles of the distribution function of maximum values of PGA for future 475 years in Fig. 6, with and without

encountering historical earthquakes in calculations. The results of study of Mousavi Bafrouei et al. (2014) are also listed in Table 1.

Table 1. Comparing 90% quantiles of distribution of PGA (in g) for two cases of including and excluding historical data and results of Mousavi Bafrouei et al (2014).

City	90% quantile of distribution of maximum value of PGA (in g) on the future time interval of the length T=475 years		PGA (in g) from Mousavi Bafrouei et al. (2014) for 475 year time period.
	Excluding historical events	Including historical events	
Bandar Abbas	0.384	0.262	0.353
Esfahan	0.046	0.036	0.112
Kerman	0.142	0.099	0.189
Rasht	0.247	0.125	0.252
Tabriz	0.138	0.400	0.354
Tehran	0.088	0.230	0.247
Yazd	0.103	0.039	0.111
Zanjan	0.143	0.085	0.191

According to Fig. 6 in all studied cities except for Tabriz and Tehran in the case of including historical data, PGA values decrease. In contrary to the conventional PSHA method where a single value is presented for PGA in each return period, in Bayesian method a distribution function is allocated to PGA in different future times. In regard to inevitable uncertainty in time and location of the earthquake, this is an advantage of the Bayesian method in prediction of events relative to PSHA conventional method.

According to Figs. 1 and 5, in Zanjan, Kerman, Yazd and Esfahan, there are insufficient data relative to active faults in these cities. Hence, the maximum estimated PGA at 90% quantile, for time interval of 475 years, with consideration of PGA value gained in the study of Mousavi Bafrouei et al. (2014) for the return period of 475 (Table 1), has been estimated less than other cities. Note that in Yazd, like other mentioned cities we also have a lack of data, but due to the occurrence of a moderate earthquake in close proximity to this city, the estimated value is higher than what they

estimated in Isfahan, Kerman and Zanjan. In other cities, considering that we have relatively sufficient data, the maximum estimated PGA amount in 90% quantile in 475 time interval has been calculated close to or greater than PSHA modified method. Table 1 and Figs 6 and 1 show that in sites where historical events are smaller than instrumental events, the PGA significantly reduced by considering historical events and getting away from the PGA estimated by the modified probabilistic method. Conversely, in inverse situations such as Tehran and Tabriz, this is quite the opposite.

In a new and different study, Khoshnava et al. (2017) presented the PGA in the northern regions of Iran applying the smoothed seismicity method which has been introduced by Frankel (1995). For return period of 475 years in Tabriz, they have come to the same result as this study and also what presented in Mousavi Bafrouei (2014), but for Tehran and Rasht they have obtained a relatively higher values.

5. Conclusions

There are different ways of incorporating seismic hazard, in this study, we aimed at estimating PGA, applying Bayesian method in the high seismically active region of Iran. According to our studies the maximum value of the $\log(\text{PGA})$ in 90% quantile is estimated in 100 years in Bandar Abbas equal to 0.3g and the corresponding minimum is in Esfahan as 0.03g. Also, we selected some special regions according to different frequencies of historical and instrumental reported events and we compared to a modified PSHA result from other studies with applying the same attenuation relationships. The comparison reveals that in regions that include enough events reported due to their active faults, the Bayesian method estimated larger values rather than the modified PSHA for a 90% quantile in 475 years, in contrary in regions with low seismicity the Bayesian method estimated lower values. Given this comparison in areas with sufficient data, this method estimates more than conventional probabilistic methods.

The results reveal that in all cities listed in Table 1 except Tehran and Tabriz, when the historical events are involved in calculations, resultant PGA values are less compared to methods based on Cornell (1968); Mousavi Bafrouei et al. (2014), Khoshnevis et al. (2017), Golar (2014) and Zare (2012). On the other hand, when historical events are not involved, the results of this method are closer to the results of the above mentioned studies. The results found in this study can be used in probabilistic seismic hazard studies of Iran.

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